## A Brief History of Pi

In each of the following select a digit $0,1, \ldots, 9$ so that the statement or formula is correct. Each digit should occur exactly once.
A) Definition: For any circle, $\mathbf{A} \times \pi=\frac{\text { Circumference }}{\text { Diameter }}$
B) In I Kings 7:23 (circa 950 BC) the value B appears to be used for $\pi$.
(Not so erroneous depending on ones interpretation.)
C) Earlier the Babylonians (1900-1680 BC) had used the ratio $2 \mathrm{C} / 8$ as a better approximation to $\pi$.
D) The Egyptians in the Rhine Papyrus (circa 1650 BC ) used $4\left(\frac{\mathbf{D}}{9}\right)^{2}$ as an approximation to $\pi$.
E) In the year $189 \mathbf{E}$, the Indiana House of Representatives passed a bill (\#246) patenting such approximations to $\pi$ with the intent of charging other states for its use. (Fortunately the bill never made it though the senate thanks to the presence of a math professor and $\pi$ remains in the public domain.)
F) De Moivre (1667-1754) found the elegant formula $e^{\mathrm{i} \pi}+1=\mathbf{F}$ which relates $\pi$, the imaginary unit, the natural $\log$ base and the additive and multiplicative identities.
G) Stirling $(1692-1770)$ proved that $n!\sim \sqrt{\mathbf{G} \pi n}(n / e)^{n}$ for large integers $n$.

The following infinite series have been used to approximate $\pi$.
H) $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\ldots=\frac{\pi}{\mathbf{H}}$ (Gregory \& Leibnitz, 17th century)
I) $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\ldots=\frac{\pi^{2}}{\mathbf{I}} \quad$ (Euler, 1735)
J) $\quad \frac{1}{\pi}=\frac{2 \sqrt{2}]}{\mathrm{J} 801} \sum_{k=0}^{\infty} \frac{(4 k)!(1103+263 \mathrm{~J} 0 \mathrm{k})}{(k!)^{4}(3 \mathrm{~J} 6)^{4 k}} \quad$ (Ramanujan, 1909)

Series such as Ramaujan's in which each term appears complicated but for which the convergence is rapid have enabled pi to be computed to more than one billion places.

