

# Rafe Jones

rfjones@carleton.edu  
http://www.people.carleton.edu/~rfjones/  
(507) 222 - 4450

Carleton College  
Department of Mathematics  
1 North College St.  
Northfield, MN 50557

---

---

## Employment

CARLETON COLLEGE, Northfield, MN

*Assistant Professor*, 2012 – 2016  
*Associate Professor*, 2016 – present

COLLEGE OF THE HOLY CROSS, Worcester, MA

*Assistant Professor*, 2008 – 2012

UNIVERSITY OF WISCONSIN, Madison, WI

*VIGRE Van Vleck Visiting Assistant Professor*, 2005 – 2008

## Education

BROWN UNIVERSITY, Providence, RI

*Ph.D., Mathematics*, May 2005  
Advisor: Joseph Silverman  
Thesis Title: Galois Martingales and the Density of the p-adic Hyperbolic Mandelbrot Set

ÉCOLE NORMALE SUPÉRIEURE, Paris, France

Visiting student, Département de Mathématiques et Applications, 1998-99

AMHERST COLLEGE, Amherst, MA

*B. A., Mathematics and French*, June 1998  
Summa Cum Laude, with Departmental Distinction in Mathematics  
Phi Beta Kappa

## Research Interests

- Galois theory and irreducibility of polynomials (particularly involving iterated morphisms)
- Discrete dynamics (particularly iteration of rational functions over global and finite fields)
- Automorphism groups of rooted trees and iterated monodromy groups
- Elliptic curves and torsion fields
- Recurrence sequences

## Refereed Publications

1. Rafe Jones, Jordan Cahn, and Jacob Spear. Perfect powers in orbits of rational functions: cases of an arithmetic dynamical Mordell-Lang-type conjecture. Submitted.
2. Rafe Jones and Alon Levy. Eventually stable rational functions. *Int. J. Number Theory* **13** (9) (2017), 2299-2318.
3. Benjamin Breen, Rafe Jones, Thomas Occhipinti, and Michelle Yuen. Wild ramification in a family of low-degree extensions arising from iteration. *JP J. Algebra Number Theory Appl.* **37**(1) (2015), 69-104.
4. Spencer Hamblen, Rafe Jones, and Kalyani Madhu. The density of primes in orbits of  $z^d + c$ . *Int. Math. Res. Not. IMRN* **7** (2015) 1924–1958.

5. Rafe Jones. Fixed-point-free elements of iterated monodromy groups. *Trans. Amer. Math. Soc.* **367(3)** (2015) 2023-2049.
6. Robert Benedetto, Patrick Ingram, Rafe Jones, and Alon Levy. Attracting cycles in  $p$ -adic dynamics and height bounds for post-critically finite maps. *Duke Math. J.* **163(13)** (2014) 2325-2356.
7. Rafe Jones and Michelle Manes. Galois theory of quadratic rational functions. *Comment. Math. Helv.* **89(1)** (2014), 173-213.
8. Katharine Chamberlin, Emma Colbert, Sharon Frechette, Patrick Heffernan, Rafe Jones, and Sarah Orchard. Newly reducible iterates in families of quadratic polynomials. *Involve: A Journal of Mathematics* **5(4)** (2012), 481-495.
9. Rafe Jones. An iterative construction of irreducible polynomials reducible modulo all primes. *J. Algebra* **369** (2012), 114-128.
10. Nigel Boston and Rafe Jones. Settled polynomials over finite fields. *Proc. Amer. Math. Soc.* **140(6)** (2012), 1849-1863.
11. Rafe Jones. Achievement sets of sequences. *Amer. Math. Monthly* **118(6)** (2011), 508-521.
12. Rafe Jones and Han Peters. Blocks of monodromy groups in complex dynamics. *Geom. Dedicata* **150(1)** (2011), 137-150.
13. Rafe Jones and Jeremy Rouse (with an appendix by Jeffrey D. Achter). Galois theory of iterated endomorphisms. *Proc. Lond. Math. Soc.* **100(3)** (2010), 763-794.
14. Xander Faber, Benjamin Hutz, Patrick Ingram, Rafe Jones, Michelle Manes, Thomas J. Tucker, and Michael Zieve. Uniform bounds on pre-images under quadratic dynamical systems. *Math. Res. Lett.* **16(1)** (2009), 87-101.
15. Nigel Boston and Rafe Jones. The image of an arboreal Galois representation. *Pure Appl. Math. Q.* **5(1)** (*Special Issue: In honor of Jean-Pierre Serre, Part 2 of 2*) (2009) 213-225.
16. Rafe Jones. The density of prime divisors in the arithmetic dynamics of quadratic polynomials. *J. Lond. Math. Soc.* (2) **78(2)** (2008), 523-544.
17. Rafe Jones. Iterated Galois towers, their associated martingales, and the  $p$ -adic Mandelbrot set. *Compositio Math.* **43(5)** (2007), 1108-1126.
18. Nigel Boston and Rafe Jones. Arboreal Galois representations. *Geom. Dedicata* **124(1)** (2007), 27-35.
19. Rafe Jones and Jan Pearce. A postmodern view of fractions and the reciprocals of Fermat primes, *Mathematics Magazine* **73** (2000), 83-97 (Awarded Allendoerfer Prize for excellence in mathematical exposition by the MAA. See Fellowships and Awards.) Reprinted in Arthur Benjamin and Ezra Graham, editors, *Biscuits of Number Theory*, Dolciani Mathematical Expositions number 34, pages 23-38. Mathematical Association of America, Washington, DC, 2009.

### Other Publications

1. Rafe Jones. *Enlightening Symbols: A Short History of Mathematical Notation* – A Book Review. *Math. Intelligencer* **38(2)** (2016), 85-86.
2. Rafe Jones. Galois representations from pre-image trees: an arboreal survey. *Publications Mathématiques de Besançon: Algèbre et Théorie des Nombres* (2013) special issue devoted to the proceedings of the conference “Number theory and its applications,” held at the Centre International de Rencontres Mathématiques, Luminy, France, January 2012.
3. Rafe Jones. *Numbers: A Very Short Introduction* – A Book Review. *Notices of the AMS* January 2012: 50-51 (this review furnished the inspiration for the cover of the January 2012 issue).
4. Rafe Jones. Do the math: a new way of thinking about the economy. Editorial in the *Worcester Telegram & Gazette*, February 10, 2009 and *The Providence Journal*, February 18, 2009.

5. Rafe Jones. Galois actions on rooted trees. Appears in Pro-p extensions of global fields and pro-p groups. Abstracts from the workshop held May 21–27, 2006. Organized by Nigel Boston, John Coates, and Fritz Grunewald. *Oberwolfach Rep.* **3** (2006), no. 2, 1463—1535.
6. Rafe Jones. Ninety-one years [Poem]. *New Millennium Writings* 14 (June 2004): 180-81.
7. Rafe Jones. *In Code: A Mathematical Journey* – A Book Review. *Notices of the AMS* April 2003: 460-461.
8. Rafe Jones. A summer at discovery.com. *Notices of the AMS* June/July 2002: 694-696.
9. Rafe Jones. Stern perfection: mathematics as a fine art. *The Catalyst Special Edition*, Brown University, 2002.

### **Grants**

NSF Grant DMS-0852826. Title: “Arboreal Galois representations and applications to arithmetic dynamics,” \$84,353, August 20, 2008 – July 31, 2011. (Extended to July 31, 2012)

NSA Young Investigators. Title: “Arboreal Galois representations and applications to arithmetic dynamics,” August 2008-August 2010. Grant awarded but declined due to rules against holding NSA and NSF grants concurrently.

### **Fellowships and Awards**

- Project NExT National Fellow, 2008-2009
- Mathematics Department Outstanding Teaching Award, Brown University, December 2004
- Dissertation Fellowship, Brown University, Fall 2003
- VIGRE Graduate Trainee, Brown University, Spring 2002 and Spring 2003
- American Association for the Advancement of Science/American Mathematical Society Mass Media Fellowship; host site was Discovery.com in Bethesda, MD. Summer 2001.
- Allendoerfer Prize for mathematical exposition. Awarded by the Mathematical Association of America for the paper "A Postmodern View of Fractions and the Reciprocals of Fermat Primes," June 2001
- University Fellowship, Brown University, Fall 1999-Spring 2000

### **Recent Presentations and Conferences**

Portland State University Maseeh Mathematics and Statistics colloquium, October 2017, invited speaker.

Washington and Lee University colloquium, March 2017, invited speaker.

American Institute of Mathematics workshop on “The Galois theory of orbits in arithmetic dynamics” Palo Alto, CA. Leadoff speaker. May 2016.

AMS Special Session, “Arithmetic Dynamics”, Joint Mathematics Meetings, Seattle, WA. Invited speaker. January 2016.

University of Rochester Number Theory seminar, November 2015, invited speaker.

Silvermania (A conference in honor of Joseph Silverman’s 60<sup>th</sup> birthday), August 2015, invited speaker.

Brown University Algebra Seminar, March 2015, invited speaker.

Front Range Algebraic Geometry and Number Theory seminar, Colorado State University, September 2014, invited speaker.

American Institute of Mathematics workshop on “Post-critically finite maps in complex and arithmetic dynamics,” Palo Alto, CA. Invited participant. March 2014.

St. Olaf undergraduate seminar, December 2013, invited speaker.

Laboratoire Mathématiques de Besançon, conference titled “Algebraic and explicit methods in number theory.” Invited to give a 60-minute talk. September 2013.

Banff International Research Station, conference titled “The art of iterating rational functions over finite fields,” invited participant. March 2013.

### **Professional Service**

Co-organizer (with Joseph Silverman and Michelle Manes) of American Institute of Mathematics weeklong program “The Galois theory of orbits in arithmetic dynamics”, Palo Alto, CA, May 2016.

Allendoerfer Prize committee, Mathematical Association of America (2015-present)

Co-organizer (with Colleen Duffy) of AMS Special Session on Number Theory at AMS Central Sectional, Eau Claire, WI, September 2014.

Co-organizer (with Farshid Hajir and Michael Bush) of AMS Special Session on Number Theory, Arithmetic Topology, and Arithmetic Dynamics at AMS Eastern Sectional, Worcester, MA, April 2011.

Co-organizer (with Joseph Silverman and Michelle Manes) of AMS special session Arithmetic and Non-archimedean Dynamics at Joint Mathematics Meetings, San Francisco, CA, January 2010.

Reviewer for NSA grant proposals, 2006-present (4 proposals).

Co-organizer of Project NEXT session "Putting together pre-tenure review materials," Mathfest, Portland, OR, 2009.

Reviewer for Math Reviews, 2009-present (22 article reviews, 2 book reviews).

Reviewer for Zentralblatt Math, 2008-2010 (5 reviews).

Referee for the following publications (2007-present):

- Proceedings of the American Mathematical Society (6 papers)
- American Mathematical Monthly (3 papers)
- Bulletin of the London Mathematical Society (3 papers)
- Acta Arithmetica (2 papers)
- Involve (2 papers)
- Journal de Théorie des Nombres de Bordeaux (2 papers)
- Journal of Number Theory (2 papers)
- Canadian Journal of Mathematics (1 paper)
- Canadian Mathematical Bulletin (1 paper)
- Crelle’s Journal (1 paper)
- Duke Mathematical Journal (1 paper)
- Experimental Mathematics (1 paper)
- Glasgow Mathematical Journal (1 paper)
- Integers (1 paper)
- LMS Journal of Computation and Mathematics (1 paper)
- Mathematische Zeitschrift (1 paper)
- New York Journal of Mathematics (1 paper)
- Research in Number Theory (1 paper)
- Topological Methods in Nonlinear Analysis (1 paper)

### **Research Mentoring**

*Research resulting from a comps project, summer 2014, “Wild ramification in a family of low-degree extensions arising from iteration.”*

- In the winter and spring of 2014, I advised Hanan Abo Sakr, Trey Brademan, Ben Breen, and Michelle Yuen on a comps project exploring the ramification properties of the prime 2 in extensions of  $\mathbb{Q}$  generated by the roots of the second iterate of the map  $f(x) = x^2 + c$ , where  $c$  is an integer. These properties hadn’t been studied before, and are of interest to the research community. The students obtained a partial classification of this behavior as part of their project. Ben and Michelle expressed an interest in continuing

to work on the problem, and in writing up the results for publication. I enlisted Tommy Occhipinti, one of our visiting faculty members who is also a number theorist, to help out. Together we worked through this surprisingly thorny question – just stating the main result takes nearly a full page!

*Informal project advisor, University of Minnesota-Duluth REU, 2014, “On arboreal Galois representations of rational functions.”*

- At the request of the REU direction, Joseph Gallian, I proposed some possible projects before the summer began. Joe then selected this one for the student Ashvin Swaminathan, and I corresponded at length with him about his results, his proofs, and where he could look for additional problems and research directions. I read several drafts of his paper, which is currently submitted to the Journal of Algebra.

*Summer research project, 2013: “Perfect powers in orbits of rational functions.”*

- I worked with Carleton rising sophomores Jordan Cahn and Jacob Spear on a project studying under what conditions a rational function with coefficients in a number field can have an orbit containing infinitely many  $m$ th powers, for a fixed  $m$  greater than 1. This precise question evolved over discussions with Jordan and Jacob, after we started with a somewhat different and much more limited question. The students then pushed hard to find a solution to the most general form of the question, with impressive results. We are working on a joint article containing these results, which will be submitted for publication.

*Summer research project 2012: “Newly reducible iterates in families of quadratic polynomials.”*

- This summer research project involved three Holy Cross rising seniors, Emma Colbert, Patrick Hefferman, and Sarah Orchard, who continued work begun by Katherine Chamberlin in her senior thesis. Prof. Sharon Frechette co-advised the students for a portion of the summer. The three students set out to prove that for any fixed  $n$  greater than 1, there are only finitely many choices for  $b$  such that the polynomial  $f(x) = x^2 + b$  has its  $n$ th iterate irreducible, but its  $(n+1)$ st iterate reducible. This result answered a natural question arising from a 2000 paper of Danielson and Fein that appeared in the Proceedings of the American Mathematical Society. Sharon and I worked with the students to write up their results, and the paper appeared in the journal *Involve*.

*Senior Thesis, 2011-2012: “Irreducibility of iterates of a second degree polynomial,” by Katherine Chamberlin.*

- The project involves studying irreducibility properties of iterates of polynomials of the form  $f(x) = x^2 + b$ , where  $b$  is a rational number. For instance, it is not known for which values of  $b$  the third iterate of  $f$  is reducible, but the second iterate is irreducible. Completing the project involved understanding field extensions, the norm map, theorems about rational points on curves, methods for finding rational points on hyperelliptic curves, and perhaps computations involving Grobner bases. Kate gave a poster presentation on her work at the 2012 Joint Mathematics Meetings.

*University of Georgia VIGRE Summer School in Arithmetic Dynamics, May 2011*

- As one of the plenary speakers at this summer school, I designed several projects for the graduate students in attendance to work on, and consulted with them daily for a week to help their progress. The projects involved areas of currently active research.

*Senior Thesis, 2009-2010: “The proportion of  $k$ -cycles for polynomials modulo primes,” by Jonathan Root.*

- This project that involved understanding current research literature and proving two new results, one in group theory and one involving the irreducibility of certain polynomials. Jonathan wrote up his work in the form of a 15-page paper. He has also presented his results in a seminar talk at Holy Cross and another in the Brown University graduate student seminar. In addition to providing the idea for the project and mathematical guidance, I gave feedback on numerous drafts of both the thesis and presentation. Jonathan received the Crompton Gold Medal for his project, which is awarded annually to a Holy Cross student for the best scientific essay or research paper.

*Arizona Winter School, March 2010.*

- Assisted on several projects having to do with Joseph Silverman's lectures on arithmetic dynamics. One project on which I worked particularly closely has led to the manuscript "Graph components and dynamics over finite fields" by Ryan Flynn and Derek Garton, which has been submitted to the *Proceedings of the American Mathematical Society*. In addition to the work at the Winter School, I read and gave comments on several versions of the manuscript.

*University of Wisconsin Research Experience for Undergraduates (REU) in Number Theory, Summer 2006.*

- Designed and advised students on two eight-week projects dealing with arithmetic dynamics: one on primitive prime divisors in quadratic arithmetical dynamical systems, and one on Galois groups of iterates of a special family of polynomials. The former project resulted in Brian Rice's paper "Primitive prime divisors in polynomial arithmetic dynamics," *Integers* **7(1)** (2007), A26. The latter led to a paper by Richard Gottesman and Kwokfung Tang, "Quadratic recurrences with a positive density of prime divisors," *International Journal of Number Theory* **6(5)** (2010), 1027-1045.

## Teaching Experience

### At Carleton

- *Elliptic Curves (Math 395), Spring 2017*
- *Calculus II (Math 121), Fall 2012, Winter 2013, Winter 2014, Fall 2014, Winter 2017*
- *Multivariable Calculus (Math 211), Winter 2015, Fall 2017*
- *Linear Algebra (Math 232), Winter 2013, Winter 2014, Winter 2015, Spring 2015*
- *Mathematical Structures (Math 236), Spring 2014, Winter 2017*
- *Number Theory (Math 312), Spring 2013, Fall 2014*
- *Algebraic Number Theory (Math 395), Winter 2014*

### Additional courses, taught at other institutions

- *Calculus I*
- *Abstract Algebra I and II*
- *Linear Algebra and Differential Equations*
- *Introduction to Discrete Math*

## Teaching-related Activities

### The Delta Center for the Integration of Research, Teaching, and Learning, University of Wisconsin

- Completed a three week immersive course entitled "the college classroom", Summer 2007. Assignments included micro-teaching presentation, readings on mathematics pedagogy, and constructions of various assessment mechanisms.

### The Sheridan Center for Teaching and Learning in Higher Education, Brown University

*Graduate Teaching Fellow, 2004-2005*

- Served as one of two graduate fellows for the Center, which involved organizing and running various teacher-training programs for graduate students on campus, including micro-teaching sessions, a New TA Orientation, the Sheridan Teaching Seminar and Certificate Program, and a Summer Syllabus Workshop.

*Teaching Consultant, 2004-2005*

- Observed graduate student lectures, discussion sections, and labs, and provided feedback and constructive criticism based on the observee's written goals and objectives for the session.

*Sheridan Center Teaching Certificate I, II, and III*

- Received these certificates after completing certain requirements, which include participation in the Sheridan Teaching Seminar and related workshops, a departmental micro-teaching session, and an individual teaching consultation.

### **Departmental and College Service at Carleton**

Carleton Junior Faculty Affairs Committee, 2013-2015.

Carleton Faculty Compensation Committee, 2017-present

Co-organizer of the Carleton math department problem-solving group, 2012-present.

Carleton Tour of Math organizer, 2014.