Note: the exam will have a section of true-false questions, like the one below.

1. True or False. Briefly explain your answer. An incorrectly justified answer may not receive full (or any) credit.
   (a) Suppose that as a car brakes, its deceleration is proportional to the square of its velocity. Then its motion is described by the differential equation
   \[ \frac{dv}{dt} = \frac{k}{v^2}, \]
   where \( v(t) \) is the car’s velocity at time \( t \) and \( k \) is a constant.
   (b) In part (a), the constant \( k \) is positive.
   (c) Suppose that for the series \( \sum_{n=1}^{\infty} a_n \), the sequence of terms \( a_n \) satisfies \( \lim_{n \to \infty} a_n = 0 \). Then \( \sum_{n=1}^{\infty} a_n \) might converge, or might diverge; there is not enough information to tell.
   (d) If \( f(x) \) is a function with \( f(0) = 0 \), \( f'(0) = 1 \) and \( f''(0) = -2 \), then its second Taylor polynomial at \( a = 0 \) is \( T_2(x) = x - 2x^2 \).
   (e) Let \( T_2(x) \) and \( T_4(x) \) be the degree 2 and 4 MacLaurin polynomials for a function \( f(x) \). Then \( T_4(2) \) is always a better approximation to \( f(2) \) than \( T_2(2) \).

2. For \( f(x) = xe^x \), find the fourth Taylor polynomial \( T_4(x) \) at \( a = 0 \). Then use it to approximate \( -\frac{1}{e} \).

3. Solve the initial value problem
   \[ \frac{dy}{dx} = \frac{y}{x}; \quad y(1) = 3 \]

4. Solve the initial value problem
   \[ \frac{dy}{dx} = \frac{y^2}{x^2 + 1}; \quad y(0) = 1 \]

5. Determine whether these series converge. If a series converges and is geometric, find its sum.
   \[ a) \sum_{n=1}^{\infty} (2 + (-1)^n) \]
   \[ b) \sum_{n=1}^{\infty} \left( \frac{\pi}{e^n} \right) \]
   \[ c) \frac{8}{9} - \frac{16}{27} + \frac{32}{81} - \frac{64}{243} + \cdots \]