Note: the exam will have a section of true-false questions, like the one below.

1. True or False. Briefly explain your answer. An incorrectly justified answer may not receive full (or any) credit.
   (a) It’s possible for a power series to converge for all $x$ in $(1, 2]$ and for $x = 0$ but not for any other value of $x$.
   (b) Denote by $g(x)$ the twentieth derivative of $f(x) = xe^{-x^4}$. Then $g(0) = 0$.
   (c) Suppose that for the series $\sum_{n=1}^{\infty} a_n$, the sequence of terms $a_n$ satisfies $\lim_{n \to \infty} a_n = 0$. Then $\sum_{n=1}^{\infty} a_n$ might converge, or might diverge; there is not enough information to tell.
   (d) To evaluate $\int \frac{2x^2 - 2}{x^3 - 3x} \, dx$, you must use partial fractions.
   (e) The Taylor series of any function $f(x)$ at $x = 0$ converges for all values of $x$.

2. Find the following integrals:
   (a) $\int \ln x \, dx$
   (b) $\int_{1}^{\infty} \frac{\ln x}{x} \, dx$
   (c) $\int \frac{x^3}{\sqrt{1-x^2}} \, dx$

3. Solve the initial value problem
   $$\frac{dy}{dx} = \frac{y^2}{x^2 + 1}, \quad y(0) = 1$$

4. The integral $\int_{0}^{1} e^{-x^3} \, dx$ cannot be evaluated exactly. Use a method from the course this term to approximate this integral to within an error of at most $1/60$. Leave your answer as a fraction. (So select a method where finding the error bound won’t be difficult)

5. Describe another method for approximating the integral from the previous problem, and write down (but do not evaluate) a sum of five numbers that is an approximation of this integral.

6. Use ideas from the course to approximate $e^2$ to within 0.1. Leave you answer as a fraction, but explain why your approximations are correct to within 0.1.
7. Determine whether these series converge. If a series converges and is geometric, find its sum.

a) \[ \sum_{n=2}^{\infty} \frac{2^n}{n^2 - 1} \]

b) \[ \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1} \]

c) \[ \sum_{n=1}^{\infty} \frac{n!}{n^n} \text{ [Hint: it’s helpful to write } (n + 1)^{n+1} \text{ as } (n + 1)^n(n + 1).] \]

d) \[ \sum_{n=2}^{\infty} \frac{1}{n - \ln n} \]

8. Determine whether this series converges absolutely, converges conditionally, or diverges:
\[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}. \]

9. Determine the interval of convergence of the power series \[ \sum_{n=1}^{\infty} \frac{(x + 1)^n}{n^23^n}. \]