Statistics 215 Random Variables and Probability Models

Here are some useful formulas.

Expected Value	μ	$E[X] = \sum xP(X = x)$
Variance	σ^2	$\text{Var}[X] = \sum(x - \mu)^2 P(X = x)$
Standard Deviation	σ	$\sqrt{\text{Var}[X]}$

I. The Carleton Help Desk has developed the following model for the number of calls it receives per hour.

Calls	0	1	2	3
Probability	0.1	0.3	0.4	0.2

(a) How many calls should the Help Desk expect per hour?

(b) What is the standard deviation?

How many calls should the Help Desk expect over two hours? Over 24 hours? To answer these questions we need to work with the expected value and standard deviation of sums of random variables. If X and Y are random variables then

$$E[X + Y] = E[X] + E[Y].$$

If the random variables are also *independent*, then

$$\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y],$$

and thus

$$\text{SD}[X + Y] = \sqrt{\text{Var}[X + Y]} = \sqrt{\text{Var}[X] + \text{Var}[Y]}.$$

When we are dealing with more than random variables it is helpful to use subscripts. So let X_1 be the number of calls received during the first hour, let X_2 be the number of calls received during the second hour, and so on. Then $X_1 + X_2$ is the number of calls received over the two hour period. And $X_1 + X_2 + \cdots + X_{24}$ is the number of calls received over 24 hours.

The results about the Expected Value and Variance of two random variables extend to more than two random variables. So

$$E[X_1 + \cdots + X_n] = E[X_1] + \cdots + E[X_n]$$

and if all the random variables $X_1, \ldots, X_n$ are independent, then

$$\mathrm{Var}[X_1 + \cdots + X_n] = \mathrm{Var}[X_1] + \cdots + \mathrm{Var}[X_n],$$

from which it follows that

$$\mathrm{SD}[X_1 + \cdots + X_n] = \sqrt{\mathrm{Var}[X_1] + \cdots + \mathrm{Var}[X_n]}.$$

(c) How many calls can the Help Desk expect to receive from 8-10 am?

(d) What's the standard deviation for the number of calls that the Help Desk receives over this two-hour period?

(e) How many calls can the Help Desk expect to receive in 24 hours?

(f) What's the standard deviation for the number of calls over this 24 hour period?

(g) What about over a year's time? (365 days $\times$ 24 hours) What's the expected value and standard deviation for the year?

II. Is it a gamble or is it a business?

A roulette wheel has 38 slots. Eighteen slots are red, 18 are black, and two are green. It costs \$1 to make a bet on red. If you win you double your money. If you lose, the casino makes \$1. Here is a probability model for the casino's winnings after one bet on RED at the roulette table.

Winnings (\$)	-1	1
Probability	18/38	20/38

(b) Find the expected value and standard deviation of the casino's winnings. How much, on average, does the casino make every time you play red? Express your answers in dollars and cents.

(c) On average, how much does the casino make after n plays? Fill in the following table

Number of plays	Casino's average winnings	Give or take (standard deviation)
1		
2		
10		
100		\$9.99
10,000	\$526.32	
1,000,000		

Is it a gamble or is it a business?