The Tableaux Graph of a Permutation

Eric S. Egge

Carleton College

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(joint work with Kristina Garrett, St. Olaf College)

Tableaux (Ferrers Fillings)

Definition

A *tableaux* is a Ferrers diagram filled with 0s and 1s, in which each column contains at least one 1.

0	1	0	1
1	1	1	
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1			

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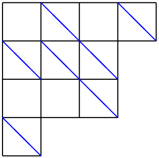

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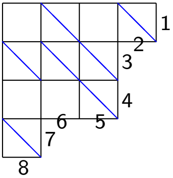

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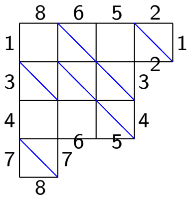

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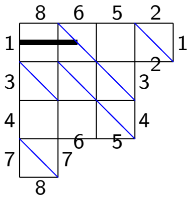

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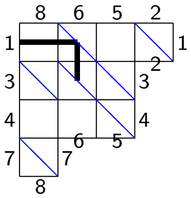

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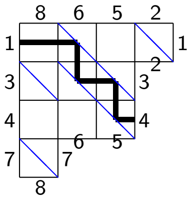

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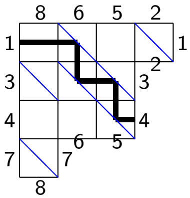

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Flips

$$\begin{array}{ccccc} a & 0 & 0 & 0 & 1 \\ 0 & & & & 0 \\ 0 & & & & 0 \\ 1 & 0 & 0 & 0 & b \end{array} \quad \longleftrightarrow \quad \begin{array}{ccccc} 1-a & 0 & 0 & 0 & 1 \\ 0 & & & & 0 \\ 0 & & & & 0 \\ 1 & 0 & 0 & 0 & 1-b \end{array}$$

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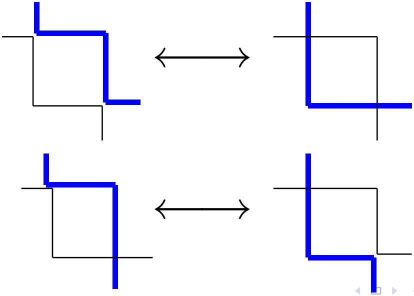
flips change tableaux but not their permutations



$\pi :=$ a permutation

vertices of $\Gamma(\pi) :=$ all tableaux mapping to π

edges of $\Gamma(\pi) :=$ pairs of tableaux differing by one flip

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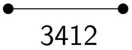

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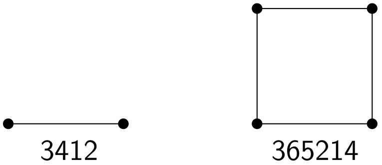

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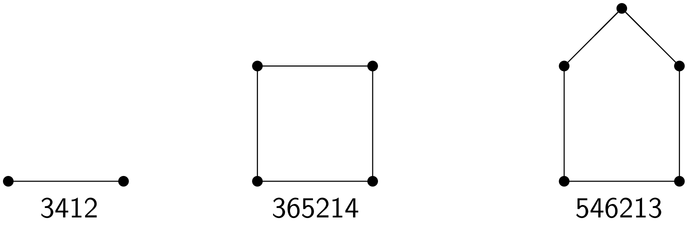

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$$21 \oplus 312 = 21534$$

$G \times H :=$ Cartesian product of G and H

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The d-dimensional hypercube is $\Gamma(\pi)$ for some $\pi \in S_{4d}$.

Conjectures About $\Gamma(\pi)$

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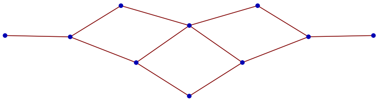
Conjecture

For any permutation π,

$$\chi(\Gamma(\pi)) \leq 3.$$

Pretty Pictures of $\Gamma(\pi)$



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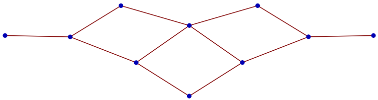

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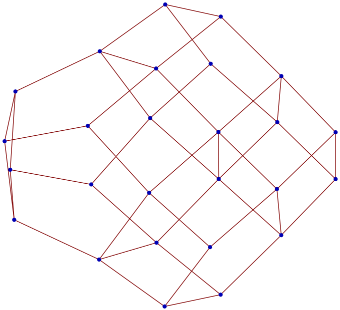

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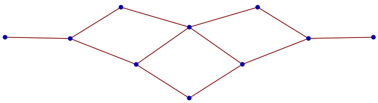

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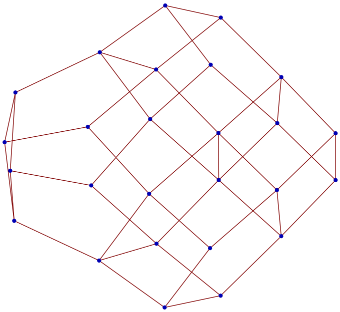

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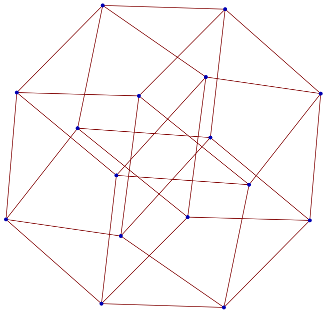

3 4 1 2 7 8 5 6 11 12 9 10 15 16 13 14