ADVANCED REVIEW

Computational techniques for parameter estimation of gravitational wave signals

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Abstract

Since the very first detection of gravitational waves from the coalescence of two black holes in 2015, Bayesian statistical methods have been routinely applied by LIGO and Virgo to extract the signal out of noisy interferometric measurements, obtain point estimates of the physical parameters responsible for producing the signal, and rigorously quantify their uncertainties. Different computational techniques have been devised depending on the source of the gravitational radiation and the gravitational waveform model used. Prominent sources of gravitational waves are binary black hole or neutron star mergers, the only objects that have been observed by detectors to date. But also gravitational waves from core-collapse supernovae, rapidly rotating neutron stars, and the stochastic gravitational-wave background are in the sensitivity band of the ground-based interferometers and expected to be observable in future observation runs. As nonlinearities of the complex waveforms and the high-dimensional parameter spaces preclude analytic evaluation of the posterior distribution, posterior inference for all these sources relies on computer-intensive simulation techniques such as Markov chain Monte Carlo methods. A review of state-of-the-art Bayesian statistical parameter estimation methods will be given for researchers in this cross-disciplinary area of gravitational wave data analysis.

This article is categorized under:

    Applications of Computational Statistics > Signal and Image Processing and Coding
    Statistical and Graphical Methods of Data Analysis > Markov Chain Monte Carlo (MCMC)
    Statistical Models > Time Series Models

KEYWORDS

Bayesian inference, Markov chain Monte Carlo, Nested Sampling, parameter estimation, signal processing

1 | INTRODUCTION

The era of observational gravitational-wave astronomy truly began with the detection of GW150914—gravitational waves from the inspiral and merger of two stellar-mass black holes that coalesced to form a single rotating black hole—by the two Advanced Laser Interferometer Gravitational-wave Observatory (Advanced LIGO) detectors (B. Abbott et al., 2016a) in 2015. Even though the existence of gravitational waves had already been predicted by Einstein (1916) as a consequence of the theory of General Relativity, only indirect evidence had so far been provided by radio observations of the binary pulsar PSR1913+16 in 1974 and the double pulsar PSR J0737-3039 in 2003. Taylor and Weisberg (1982) showed that the energy loss associated with the orbital decay rate was consistent with the emission of gravitational waves. The discoverers of PSR1913+16 (Hulse & Taylor, 1975), Joseph Taylor and Russell Hulse, were awarded the Nobel Prize in Physics in 1993. The groundbreaking first direct measurement of gravitational waves earned the founders of LIGO, Rainer Weiss, Barry Barish, and Kip Thorne, the Nobel Prize in Physics in 2017. Since then, many more black hole mergers have been observed in subsequent observation runs of the Advanced LIGO and Virgo interferometers (B. Abbott et al., 2019d; R. Abbott et al., 2020; B. Abbott et al., 2020b, 2020c). The detection of the neutron star merger GW170817 (B. Abbott et al., 2017c), seen both in gravitational and electromagnetic waves, ushered in the new era of gravitational-wave multi-messenger astronomy (B. Abbott et al., 2017d).

Gravitational waves are produced by non-axisymmetric acceleration of mass, such as two compact neutron stars orbiting each other. They are quadrupole (lowest order) waves that propagate outwards from their source at the speed of light. Unlike electromagnetic waves, they pass through any matter. Their effect is orthogonal to the direction of propagation. They have two polarizations, a plus- and cross- polarization. The plus-polarization stretches the distance of two points on the horizontal axis and simultaneously compresses the distance between two points on the vertical axis. The cross-polarization has a similar effect, but rotated by 45°. Gravitational wave detectors, such as Advanced LIGO (Aasi et al., 2015) and Advanced Virgo (Acernese et al., 2015), are based on Michelson laser interferometry (Hariharan, 2007; Pitkin, Reid, Rowan, & Hough, 2011). A beam of light is split in two and sent in two equal-length (4 km for LIGO, 3 km for Virgo) perpendicular arms (in vacuum) with mirror-coated test masses suspended on pendulums at each end of the arms, storing the light and increasing the effective arm length by a factor of ∼300 as the light bounces back and forth. The light beams exiting the arms are recombined. A passing gravitational wave changes the lengths of each arm and thus the interference pattern measured by photodetectors. The wave amplitude, the dimensionless *strain*, denoted by h, is measured by the relative change in spacing $\frac{\Delta L}{L}$ between two test masses where L denotes the equilibrium spacing. The strain is proportional to the second time derivative of the source quadrupole moment and decreases in proportion to the inverse distance from the source (Schutz, 2009). Thus, in practice, only gravitational waves from massive and rapidly accelerating objects in the Universe will be detectable. Gravitational waves from astronomical sources, due to their large distance from Earth, have detectable strains of the order of 10^{-21}.

Here, we focus on observations from the network of ground-based interferometers. After a 5-year long upgrade, Advanced LIGO became operational in 2015 with two detectors, in Hanford, Washington and Livingston, Louisiana, and Advanced Virgo with a detector in Cascina, Italy, in 2017. A third LIGO detector is planned to be built in India (Unnikrishnan, 2013), and the Japanese underground cryogenic detector KAGRA (Somiya, 2012) is currently coming online with its commissioning activities. With a single detector, it is difficult to determine the sky location of a transient source. A worldwide network of interferometers is important to estimate the source position using timing, amplitude, and polarization information of the signal (Bizouard & Papa, 2013). The Einstein Telescope (Maggiore et al., 2020; Sathyaprakash et al., 2012) is a proposed third-generation underground cryogenic detector with 10 km arm lengths in a triangular formation. The Cosmic Explorer, a planned 40 km L-shaped detector, will greatly enhance the sensitivity due to the significantly increased arm lengths (B. Abbott et al., 2017a). Pulsar timing arrays (Dahal, 2020) are sensitive to low-frequency gravitational waves in the range of 10^{-9} to 10^{-6} Hz. Furthermore, a space-based interferometer, the so-called Laser Interferometer Space Antenna (LISA) is planned to be launched in 2034 by the European Space Agency with three satellites 2.5 million kilometer apart, forming an equilateral triangle in an Earth-trailing, heliocentric orbit (Amaro-Seoane et al., 2017). A space-based interferometer would be sensitive to frequencies from 0.1 mHz to 1 Hz. Because LIGO and Virgo are currently operating, and making detections, this review is concentrating on parameter estimation methods for these ground-based detectors. However the parameter estimation development for LISA has been long and active (Umstätter et al., 2005a, 2005b; N. J. Cornish & Crowder, 2005; Crowder & Cornish, 2007; Babak et al., 2008; Stroeer et al., 2007; Arnaud et al., 2007; Ali, Christensen, Meyer, & Rover, 2012; Baghi et al., 2019; Katz, Marsat, Chua, Babak, & Larson, 2020; Toubiana, Marsat, Babak, Baker, & Canton, 2020). LISA will actually observe thousands of simultaneous signals, and therefore pose one of the biggest parameter estimation challenge ever in physics

(Babak, 2017; Robson & Cornish, 2017; N. Cornish & Robson, 2017; T. Littenberg, Cornish, Lackeos, & Robson, 2020; Marsat, Baker, & Canton, 2020; N. Cornish & Shuman, 2020).

Even though much information about the emitting source of the gravitational waves can be learned from the direct inspection of the observed gravitational waveforms (LIGO Scientific & Virgo Collaborations, 2017), the full posterior distribution of the source parameters using Bayesian computational techniques is required to accurately estimate the properties of the source, such as the masses and spins of the two black holes, and to quantify associated uncertainties. A review of state-of-the-art parameter estimation methods will be the focus of this paper. As this is not only an expansive but also a rapidly evolving research area, we can only strive for a comprehensive review but cannot claim exhaustiveness. For details regarding the computational methodology and their implementations, we refer to the relevant statistical and machine learning literature.

2 | PARAMETER ESTIMATION

Following a calibration procedure for each detector, the time series of dimensionless strain measurements are produced. The strain is the fractional change in spacings between two test masses due to a passing gravitational wave, that is the difference in lengths relative to the total arm length of the interferometer. The calibrated observations $\mathbf{d}^{(k)} = (d^{(k)}(t))$, $t = 1, ..., T$ of gravitational waves from detector k are modeled as a deterministic signal, the strain $\mathbf{h}^{(k)} = (h^{(k)}(t|\boldsymbol{\theta}))$, $t = 1, ..., T$ plus additive interferometer noise $n^{(k)}(t)$, that is,

$$d^{(k)}(t) = h^{(k)}(t|\boldsymbol{\theta}) + n^{(k)}(t), \quad t = 1, ..., T. \tag{1}$$

Thus, the model consists of two parts: a model for the gravitational wave signal and a model for the noise, both equally important for parameter estimation (B. Abbott et al., 2020).

The interferometers are subject to a variety of *noise* components including quantum noise, seismic noise, thermal noise, and gravity gradient noise. Also, environmental noise or malfunctioning of equipment can cause transient noise events (B. Abbott et al., 2016; Nuttall, 2018), or noise spectral lines (Covas et al., 2018). All these combined are modeled by the noise time series $n^{(k)}(t)$, usually assumed to be a Gaussian, stationary time series with zero mean and covariance matrix Σ_k.

The assumptions on the noise determine the form of the likelihood. The observations from different detectors in a network of K detectors are usually assumed to be independent and thus, for a coherent analysis that includes data from all detectors, the joint likelihood is the product of the individual likelihood functions:

$$L(\mathbf{d}|\boldsymbol{\theta}) = \prod_{k=1}^{K} L^{(k)}\left(\mathbf{d}^{(k)}|\boldsymbol{\theta}\right) = \prod_{k=1}^{K} \frac{1}{\det(2\pi\Sigma_k)^{1/2}} e^{-\frac{1}{2}\left(\mathbf{d}^{(k)} - \mathbf{h}^{(k)}\right)^{\top}\Sigma_k^{-1}\left(\mathbf{d}^{(k)} - \mathbf{h}^{(k)}\right)} \tag{2}$$

Usually, parameter estimation is implemented in the frequency domain because after a Fourier transform, instead of a multivariate Gaussian likelihood with non-diagonal covariance matrix, the complex vector $\tilde{\mathbf{d}}^{(k)}$ containing Fourier coefficients defined by

$$\tilde{d}_j^{(k)} = \tilde{d}^{(k)}\left(f_j\right) = \sum_{t=1}^{T} d^{(k)}(t)e^{-itf_j}$$

at the Fourier frequencies $f_j = 2\pi j/T$, $j = 0, ..., N$, $N = \lfloor(T-1)/2\rfloor$, is approximately (for large T) a complex multivariate Gaussian but with a *diagonal* covariance matrix $S^{(k)}$ that contains the power spectral density $S^{(k)}(f_j)$ at the Fourier frequencies f_j on the diagonal. The power spectral density

$$S^{(k)}(f) = \sum_{\ell = -\infty}^{\infty} \gamma^{(k)}(\ell)e^{-i\ell f} \tag{3}$$

is the Fourier transform of the autocovariance function $\gamma^{(k)}(l)$ of the stationary noise time series. The diagonal covariance structure in this so-called *Whittle* likelihood approximation (Cutler & Flanagan, 1994; Finn, 1992; Kirch, Edwards, Meier, & Meyer, 2019; Whittle, 1957) facilitates the calculation of the inverse and determinant in the Whittle likelihood:

$$L(\mathbf{d}|\boldsymbol{\theta}) \approx \prod_{k=1}^{K} \frac{1}{\det\left(\pi T \mathbf{S}^{(k)}\right)} e^{-\frac{1}{T}\left(\tilde{\mathbf{d}}^{(k)} - \tilde{\mathbf{h}}^{(k)}\right)^* \mathbf{S}^{(k)-1}\left(\tilde{\mathbf{d}}^{(k)} - \tilde{\mathbf{h}}^{(k)}\right)} \tag{4}$$

In the following, we assume that the power spectral density (PSD) for each detector is known but in Section 7, we outline procedures to deal with an unknown noise spectral density that is estimated simultaneously with the signal parameters. The stationarity assumption is usually adequate for transient signals such as the signal from a merger of two black holes but it is well known that the interferometer noise is slowly time-varying over longer periods of about 1 min (Chatziioannou et al., 2019). Therefore, it will be important to take the time-varying noise into account when estimating the parameters of gravitational wave signals of longer duration such as those produced by neutron star mergers or pulsars. Furthermore, the power spectrum contains many high power narrow spectral lines which are not compatible with the Gaussian assumption. Many of these are due to known sources such as the 60 Hz power line harmonics, so-called "violin modes" caused by thermally excited mirror suspension and their harmonics, or calibration lines inserted by moving the end mirrors (Covas et al., 2018). Berry et al. (2015) perform a systematic study on the performance of parameter estimates under the Gaussian assumption but with real, non-ideal noise. Figure 1 shows the amplitude spectral density (the square root of the power spectral density) of both Advanced LIGO detectors and the Advanced Virgo detector from their second observational run (B. Abbott, Abbott, Abbott, Abraham, et al., 2019d). For the purpose of parameter estimation, the power spectral density is usually assumed to be known and fixed. Its value is usually obtained by one of two methods. The Welch method (Welch, 1967) uses a separate stretch of data close to but not containing the time period of the signal. This stretch of data is divided into overlapping segments of the same duration T as the signal period. These segments are Fourier transformed using the FFT algorithm after windowing with a Tukey or Hanning window to avoid spectral leakage (B. Abbott, Abbott, Abbott, et al., 2020). The Welch method averages over the periodograms of each individual time segment to reduce the variance of the estimate. Often, the median instead of mean of the periodograms is used as it is more robust with respect to outliers (Veitch et al., 2015). As an alternative to this off-source Welch method, an on-source estimate, based on the same data that contains the signal, is obtained by BayesWave (N. J. Cornish & Littenberg, 2015), described in Section 7. The power spectral density is then assumed to be known and equal to the Welch or BayesWave estimate when estimating the parameters of the signal. The likelihood Equation (4) for known PSD simplifies to

$$L(\mathbf{d}|\boldsymbol{\theta}) \propto \prod_{k=1}^{K} e^{-\frac{1}{T}\left(\tilde{\mathbf{d}}^{(k)} - \tilde{\mathbf{h}}^{(k)}\right)^* \mathbf{s}^{(k)-1}\left(\tilde{\mathbf{d}}^{(k)} - \tilde{\mathbf{h}}^{(k)}\right)} \tag{5}$$
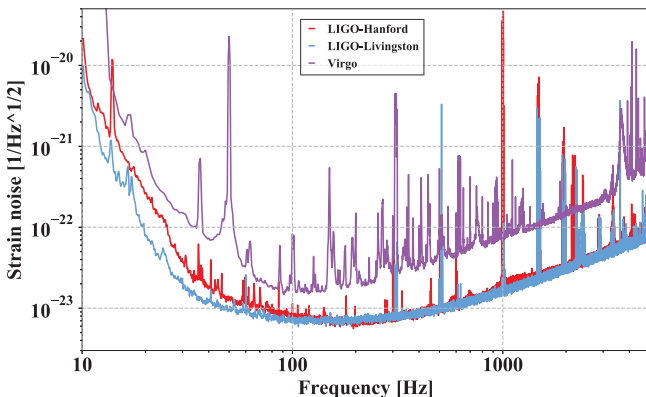

FIGURE 1 The Advanced LIGO and Advanced Virgo sensitivity curves. These are typical of the best sensitivities of the detectors during their second observational run (B. Abbott, Abbott, Abbott, Abraham, et al., 2019d).
Source: LIGO Scientific Collaboration

Alternative parametric and nonparametric Bayesian estimates of the power spectral density are discussed in Section 7.

The *strain signal* is characterized by various parameters (depending on the gravitational wave source) such as the masses and spins of the progenitors in case of a compact binary coalescence, compiled in the parameter vector $\boldsymbol{\theta} = (\theta_1, ..., \theta_p)$. Considering a geocentric reference frame, the strain measured at detector k of a gravitational wave source with polarization amplitudes h_+ and $h_\times$ located in the sky at (α, δ) where α is the right ascension and δ the declination of the source is

$$h^{(k)}(t|\boldsymbol{\theta}) = F_+^{(k)}(\alpha,\delta,\psi)h_+\left(t - \tau^{(k)}|\boldsymbol{\theta}\right) + F_\times^{(k)}(\alpha,\delta,\psi)h_\times\left(t - \tau^{(k)}|\boldsymbol{\theta}\right). \tag{6}$$

$F_{+,\times}$ are the antenna response functions that depend on the source locations, the polarization angle ψ of the waves (Anderson, Brady, Creighton, & Flanagan, 2001; Christensen, 1992), plus the locations and orientations of the detectors. For short transient signals, the time dependence of the antenna response functions due to the rotation of the earth can be safely ignored and assumed to be constant throughout the observation period, but needs to be taken into account for long signals. The parameter $\tau^{(k)} = \tau^{(k)}(\alpha, \delta)$ denotes the location-dependent time delay. For a detailed explanation of the calibration of the gravitational wave detectors and methods to take the associated calibration uncertainty into account, we refer to Sun et al. (2020), Viets et al. (2018), Cahillane et al. (2017), and Acernese et al. (2018).

Some preprocessing steps are necessary. The time series is usually downsampled from its original sampling frequency of 16,384 Hz for LIGO and 20 kHz for Virgo to a lower rate (typically 4,096 Hz). Then it is band-pass filtered because the LIGO detectors are calibrated only in the frequency band from 10 Hz to 5 kHz and the Virgo detector is calibrated in the band from 10 Hz to 8 kHz. In some analyses, the time series is also notch filtered around known instrumental noise frequencies. Software packages for preprocessing are included in the LAL library (LIGO Scientific Collaboration, 2018) and the Gravitational Wave Open Science Center (Abbott et al., 2019; LIGO Scientific Collaboration, Virgo Collaboration, 2020b). Parameter estimation is often performed in the frequency domain. After a discrete Fourier transform, model (1) is equivalent to the following frequency domain model:

$$\tilde{d}_j = \tilde{d}\left(f_j\right) = \tilde{h}^{(k)}\left(f_j|\boldsymbol{\theta}\right) + \tilde{n}^{(k)}\left(f_j\right), \ \ f_j = 2\pi j/T, \ \ j = 0,...,N \tag{7}$$

with

$$\tilde{h}^{(k)}\left(f_j|\boldsymbol{\theta}\right) = \left(F_+^{(k)}(\alpha,\delta,\psi)\tilde{h}_+(f_j|\boldsymbol{\theta}) + F_\times^{(k)}(\alpha,\delta,\psi)\tilde{h}_\times(f_j|\boldsymbol{\theta})\right)e^{-if_j\tau^{(k)}} \tag{8}$$

The exact form of the gravitational waveform model $h_{+,\times}(t|\boldsymbol{\theta})$ depends on the emitting source of the gravitational waves. In the following sections, we describe these waveform models from compact binary coalescences, burst signals from core-collapse supernovae, continuous signals from rapidly rotating neutron stars (pulsars), and stochastic signals from astrophysical and cosmological origins that combine to form the stochastic gravitational-wave background, illustrated in Figure 2. For each of these gravitational waveform sources, we will specify the computational techniques used to estimate their parameters. In Section 7, we describe methods for estimating the unknown noise spectral density in unison with the signal parameters.

3 | COMPACT BINARY COALESCENCES

To date, the Advanced LIGO and Advanced Virgo detectors have observed gravitational waves from the coalescence of dozens of compact binary systems containing black holes and also two neutron star mergers. Chirp-like signals of short duration, such as GW150914 (B. Abbott et al., 2016c), are generated during the final stage of a binary system as the two progenitor masses spiral in toward one another and then merge. Many different waveform families for binary mergers exist in the literature. Most are parametric waveform models obtained by solving Einstein's equations and can be constructed within different frameworks. They can be based on solving the two-body dynamics in general relativity perturbatively using post-Newtonian (PN) approximation of various orders (Blanchet, 2014; Buonanno, Iyer, Ochsner,
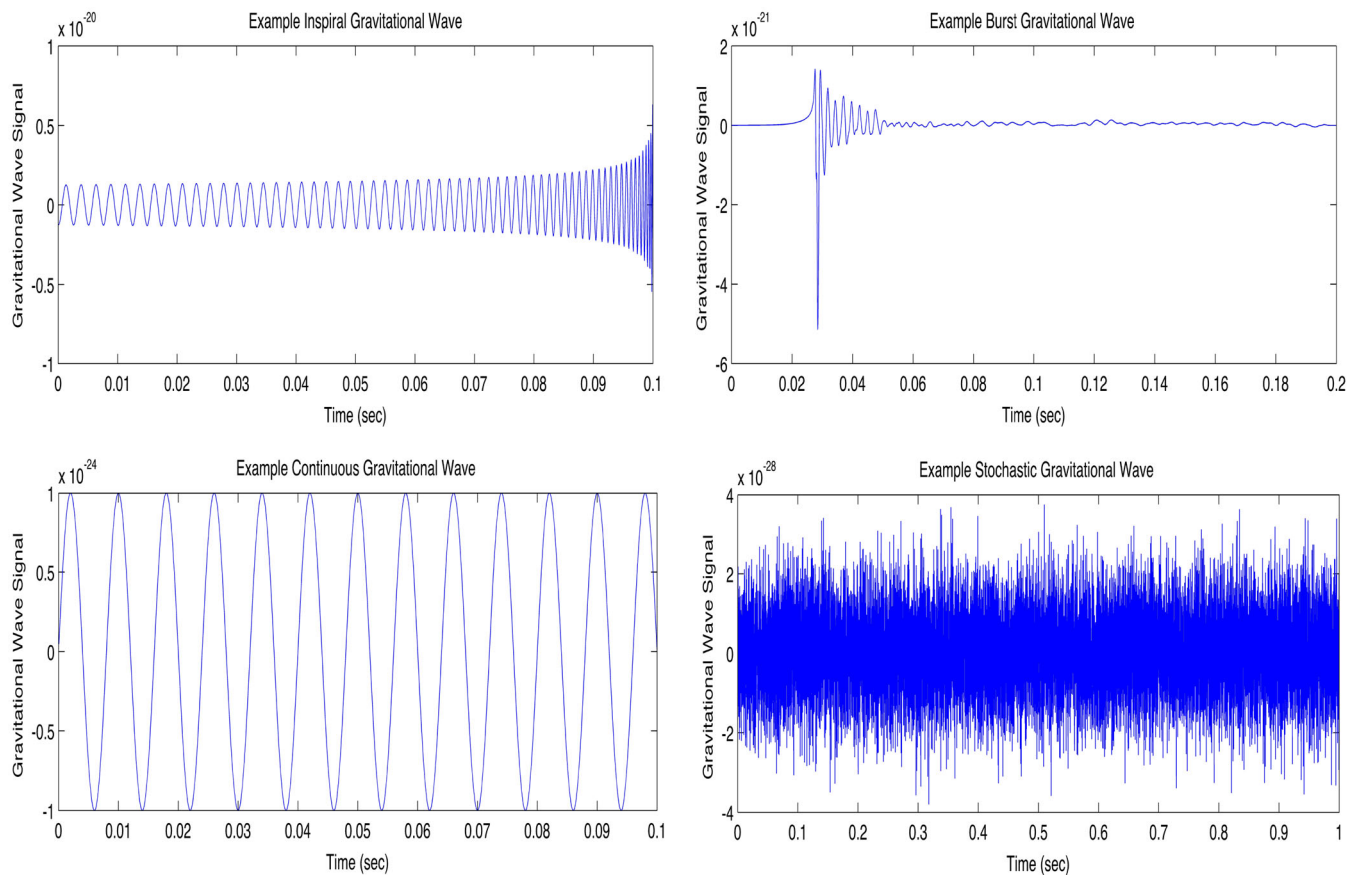
FIGURE 2 Simulated gravitational wave signals with different waveforms: a compact binary inspiral, core-collapse supernova, continuous-wave, and stochastic gravitational-wave signal. Credit: A. Stuver, LIGO Scientific Collaboration (Stuver, 2020)

Pan, & Sathyaprakash, 2009; Damour, 2016). Two other frameworks were considered for the analysis of GW150914: the effective-one-body (EOBNR) waveforms (Buonanno & Damour, 1999; Nagar et al., 2018; Ossokine et al., 2020; Pürrer, 2016) where higher-order PN terms are calibrated to numerical relativity (NR) simulations, and hybrid/phenomenological (IMRPhenom) waveforms (Hannam et al., 2014; Khan et al., 2016; Pratten et al., 2020) based on extending frequency-domain PN expressions and hybridizing PN and EOB with NR waveforms. For the simulation of gravitational wave signals, numerical relativity calculations have become very important (Brügmann, 2018; Eisenstein, 2019). Surrogate models of NR waveforms have been shown to be both fast and accurate (Varma et al., 2019).

When dealing with a binary neutron star system, or a neutron star—black hole binary, the neutron star can be distorted from its spherical shape from the tidal gravitational fields that it experiences. This deformation will actually increase the rate of the inspiral. In this case, the gravitational waveform model should address this deformation. For example, with the examination of the gravitational wave data from the binary neutron star merger GW170817 (B. Abbott, Abbott, Abbott, Acernese, et al., 2017c) the waveform used was generated with the post-Newtonian and EOBNR formalisms, which also included tidal deformations (Vines, Flanagan, & Hinderer, 2011; B. Abbott et al., 2019f).

Here we exemplify the parameter estimation using 8 s of data around the transient event GW150914 (B. Abbott et al., 2016d; Romero-Shaw et al., 2020) from both Advanced LIGO detectors, comprising about 10 cycles during the inspiral phase, followed by merger and ring-down. Merging black holes in a quasi-circular orbit are described by eight *intrinsic* parameters, the masses $m_{1,2}$ and the spins $s_{1,2}$ (in terms of the dimensionless spin magnitude $a = c|s|/(Gm^2) \in [0, 1]$ and orientation) of the individual black holes and an additional seven *extrinsic* parameters, the luminosity distance D_L, the right ascension α and declination δ of the source, the orientation in terms of the inclination angle ι between the system's orbital angular momentum and the line of sight, and the polarization angle ψ, the time t_c and the phase ϕ_c of coalescence. If the spins of the masses are parallel to the orbital plane, then there can be a spin-orbit

coupling that will cause a precession of the orbital plane. In this case, ι is not well defined, and it is more appropriate to define the angle between the total angular momentum of the system and the line of sight, θ_{JN}. We will ignore orbital precession for the remainder of this article and continue to use ι. For GW150914, the orbital eccentricity of the binary system was not considered. This would have added an additional two parameters, the magnitude and the argument of periapsis of the system.

During the inspiral phase, the gravitational wave polarizations observed at the angle ι can be expressed at the leading order as

$$h_+(t|\boldsymbol{\theta}) = A(t|\boldsymbol{\theta})\frac{1}{2}\left(1+\cos^2\iota\right)\cos\phi(t|\boldsymbol{\theta}) \tag{9}$$

$$h_\times(t|\boldsymbol{\theta}) = A(t|\boldsymbol{\theta})\cos\iota\sin\phi(t|\boldsymbol{\theta}) \tag{10}$$

where $A(t|\boldsymbol{\theta})$ and $\phi(t|\boldsymbol{\theta})$ are the gravitational wave amplitude and phase, respectively. The gravitational wave frequency f equals twice the orbital frequency. Due to the emission of gravitational waves, the binary system loses energy causing the orbital distance to decrease and the orbital frequency to increase. The phase evolution $\phi(t)$ in the inspiral regime is well described by post-Newtonian theory, a perturbative expansion in powers of the orbital velocity v/c. The first order gravitational-wave frequency evolution is described by the differential equation (LIGO Scientific & Virgo Collaborations, 2017)

$$\frac{df}{dt} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3}f^{11/3} \tag{11}$$

where c is the speed of light and G Newton's gravitational constant. We see that the phase evolution is mainly influenced by the chirp mass $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$ as a result of energy loss from emitting gravitational waves. The chirp mass can be much more accurately estimated than the individual masses. Additional parameters such as the mass ratio $q = m_1/m_2$ (or equivalently the symmetric mass ratio $\eta = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{q}{(1+q)^2}$) and the spin components enter at each of the following PN orders. This is accurate in the inspiral phase but degrades as the black holes get closer and eventually the full solution to Einstein's equations is needed using numeral relativity. Merger and ringdown depend primarily on the mass and spin of the final black hole. Note that the observed frequency of the signal is redshifted by a factor $(1+z)$ where z is the cosmological redshift (Krolak & Schutz, 1987) and cannot be distinguished from a rescaling of the masses by the same factor. Thus, the source mass is obtained by dividing the measured redshifted mass by $(1+z)$. When the luminosity distance is estimated this can be converted to a redshift z by assuming a Lambda Cold Dark Matter (ΛCDM) cosmology and an appropriate value of the Hubble constant (Ade et al., 2016).

After a Fourier transform of the strain time series, parameter estimation is performed in the frequency domain using the likelihood function (4). Note that most waveform models used in practice are constructed in the frequency domain. That includes phenomenological models, surrogate models of effective-one-body waveforms, and surrogate models built directly from NR waveforms. The Bayesian model is completed by specifying prior distributions for the parameters. For parameter estimation of GW150914, the masses were assumed to have a uniform prior on [10, 80] $\mathbf{M}_\odot$ with $m_2 \leq m_1$. The spin magnitudes $a_{1,2}$ were assumed to be uniformly distributed on (0, 1) and the spin orientations isotropic on the 2-sphere. The prior on t_c was chosen to be uniform, centered at the reported time of coalescence with a width of 0.2 s, ϕ_c and ψ were assumed to be uniform on $[0, 2\pi]$. As the density of sources is assumed uniform in the cosmological co-moving volume, the prior for the source location in the Universe was isotropic, that is, the prior for α uniform on $[0, 2\pi]$ and the prior for $\cos(\delta)$ uniform on $[-1, 1]$, and the distance prior is uniform in Euclidian volume. The prior on the cosine of the inclination angle ι was uniform on $[-1, 1]$.

The product of prior and likelihood determines the posterior distribution of the parameters using Bayes' theorem

$$p(\boldsymbol{\theta}|\mathbf{d}) = \frac{L(\mathbf{d}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{\int L(\mathbf{d}|\boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}} \tag{12}$$

A comprehensive treatment of Bayesian inference can be found in the textbook of Gelman, Carlin, Stern, and Rubin (2014) and an introduction to Bayesian inference for astronomers in Thrane and Talbot (2019). Evaluation of the normalizing constant in the denominator (also called the *marginal likelihood* or *evidence*) and calculating marginal distributions and their summary statistics of individual parameters requires high-dimensional integration. To solve these high-dimensional integration problems, computer-intensive simulation-based methods are required for several reasons: the posterior distribution is not tractable analytically, numerical integration is only feasible in low dimensions, Laplace approximation (Gelman et al., 2014) is suitable only for unimodal and symmetric posteriors, and ordinary simulation methods based on independent random draws such as importance sampling (Gelman et al., 2014) are also only applicable effectively in low dimensions. Christensen and Meyer (1998) demonstrated the use of Markov chain Monte Carlo (MCMC) methods for gravitational wave parameter estimation for posterior computation using a simple low-order waveform model with four parameters. MCMC methods were readily taken up by LIGO—Virgo and increasingly sophisticated MCMC techniques were developed to handle higher-order waveform approximations with increasing number of parameters (Aasi et al., 2013; Christensen & Meyer, 2001; Christensen, Meyer, & Libson, 2004; Pai, Dhurandhar, & Bose, 2001; Röiver, Meyer, & Christensen, 2006; Röiver, Meyer, & Christensen, 2007; van der Sluys et al., 2008; van der Sluys et al., 2008; Veitch & Vecchio, 2010). Implemented in LALInference is an adaptive version of the Metropolis–Hastings algorithm (Metropolis, Rosenbluth, Rosenbluth, Teller, & Teller, 1953) coupled with parallel tempering (Swendsen & Wang, 1986; Veitch et al., 2015) which is sufficiently flexible to handle any of the waveform models in the LAL libraries. For a detailed description of the parallel tempering MH algorithm, see van der Sluys, Röver, et al. (2008); van der Sluys, Raymond, et al. (2008).

Parallel tempering uses a series of functional probability densities also known as power posterior densities, which generate a bridge between the prior and the posterior distributions. The sampling is performed on these bridging densities, but allowing point swaps in adjacent chains, according to a certain probability. Thus, the exploration of the posterior distribution is prevented from being stuck in certain areas of the parameter space. This algorithm has been implemented in software packages such as LALinference (Veitch et al., 2015) and BayesWave (N. J. Cornish & Littenberg, 2015). Even though the samples from these intermediate bridging densities are discarded from the parameter inference process, these samples can be used to accurately calculate the marginal likelihood (e.g., T. B. Littenberg & Cornish, 2009; Veitch et al., 2015; Maturana-Russel, Meyer, Veitch, & Christensen, 2019). They can also be used to build a proposal distribution based upon estimation of the kernel density and tuned to the target posterior (Farr, Kalogera, & Luijten, 2014). The open-source Python-based parameter estimation toolkit for compact binary coalescence signals, `PyCBC Inference` (Biwer et al., 2019), uses the ensemble MCMC algorithm `emcee` (Foreman-Mackey, Hogg, Lang, & Goodman, 2013) and its parallel-tempered version `emcee_pt` (Vousden, Farr, & Mandel, 2015).

An alternative algorithm, also implemented in LALInference and routinely used together with the MCMC algorithm is nested sampling (NS; Skilling, 2006), which evaluates the evidence/marginal likelihood, that is, the denominator in Bayes theorem (Equation (12)). It can also be extended to generate samples from the posterior distribution at no extra cost. Its particular way of exploring the parameter space allows it to work in cases in which popular MCMC methods fail (Maturana-Russel, Brewer, Klaere, & Bouckaert, 2019). Basically, it does this by sampling a number of points from the prior and then the one with the lowest likelihood value is replaced by a new point drawn from the prior, but restricted to have a likelihood higher than the one that is being replaced. This procedure is repeated multiple times, allowing to keep points in different modes simultaneously and deal with complex likelihood functions. NS was first used for gravitational wave searches with ground-based observatories (Veitch & Vecchio, 2008a, 2008b). As it can generate samples from the posterior, it was then applied for parameter estimation and model selection for binary inspiral systems (Veitch & Vecchio, 2010). NS type algorithms have also been used for model selection and parameter estimation of space-based detectors (e.g., Feroz, Gair, Hobson, & Porter, 2009; Gair et al., 2010; Gair & Porter, 2009; Marsat et al., 2020). NS is available in computational packages such as Bilby (Ashton et al., 2019; Romero-Shaw et al., 2020; a flexible Python-based package that also includes several MCMC samplers) and in parallelized versions (R. Smith & Ashton, 2019), which can be used in computing clusters.

Both MCMC and NS were used to estimate the parameters of the very first gravitational wave signal GW150914 observed by Advanced LIGO, yielding consistent sets of parameter estimates (B. Abbott et al., 2016d). A reconstruction of the gravitational wave signal is shown in Figure 3. A table with parameter estimates and their standard errors can be found in table I of B. Abbott et al. (2016d). The importance that Bayesian parameter estimation played in describing the physics associated with the first direct observation of gravitational waves with GW150914 is summarized in Meyer and Christensen (2016).

FIGURE 3 The detector strain measurements of GW150914 (B. Abbott et al., 2016c) observed by the LIGO Hanford (H1, top) and Livingston (L1, bottom) detectors in gray. Times are shown relative to September 14, 2015, at 09:50:45 UTC. The cyan curve is the estimated signal using the IMRPhenom and EOBNR waveform templates, the blue curve a signal reconstruction based on BayesWave.
Source: B. Abbott et al. (2016d)
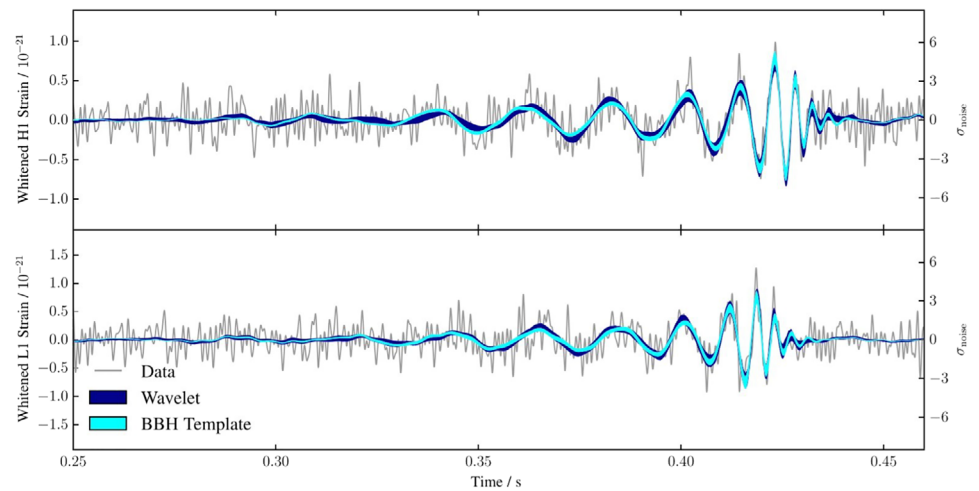
In a compact binary coalescence event that includes at least one neutron star—a binary neutron star or a neutron star—black hole merger—electromagnetic signatures at different timescales and wavelengths are expected if the neutron star is tidally disrupted before merger. Being able to rapidly provide an estimate of the sky location is particularly important for multi-messenger astronomy (B. Abbott et al., 2017d). After the detection of GW150914, the estimate of its sky location was shared with 63 ground- and space-based observatories covering radio, optical, near-infrared, X-ray, and gamma-ray wavelengths (B. Abbott et al., 2016b). Fast localization of the gravitational wave source allows a targeted follow-up by electromagnetic telescopes as was the case for GW170817. To this end, Bayestar has been developed. Bayestar conditions on fixed values of the intrinsic parameters and computes the posterior of the extrinsic parameters. This allows an approximation of the marginal posterior distribution of the sky location via numerical integration (L. P. Singer & Price, 2016). This can provide reliable sky localization and distance estimations within minutes after detection. Bayestar gave an initial estimate of the position in the sky of GW170817 of 31 deg^2 and an estimate of the luminosity distance of 40 ± 8 Mpc (LIGO Scientific Collaboration, Virgo Collaboration, 2017). A more accurate estimate of the sky location, 28 deg^2, was then provided by LALInference (B. Abbott, Abbott, Abbott, Acernese, et al., 2017c). Methods for describing the three-dimensional posterior distribution of sky location and distance have been developed (L. Singer et al., 2016; L. P. Singer et al., 2016).

A different approach to modeling the gravitational waves of compact binary coalescences is implemented in the BayesWave algorithm of N. J. Cornish and Littenberg (2015). It is not based on a physically meaningful gravitational waveform model but aims to reconstruct the shape of any burst signal using wavelets. The BayesWave reconstruction of GW150914 is displayed in Figure 3. Ghonge et al. (2020) compare its reconstruction properties to the reconstructions via MCMC and NS in LALInference.

Sampling algorithms such as MCMC and NS give generally accurate parameter estimates for compact binary inspirals but can be very slow, for example, it takes days to obtain results for black hole mergers, weeks for neutron star mergers. Efficient but accurate approximative methods (R. Smith et al., 2016; Canizares et al., 2015; L. P. Singer & Price, 2016) can yield a significant reduction in computation time. Reduced-order models (ROMs) of gravitational waveforms have reduced the computational cost of Bayesian inference by a more efficient decomposition of the waveform using analytical insight (Pürrer, 2015). R. Smith et al. (2016) construct a ROM that includes the effects of spin precession, inspiral, merger, and ringdown in compact object binaries utilizing the "IMRPhenomPv2" waveform model. A fast reduced-order quadrature allows to approximate posterior distributions at greatly reduced computational costs. A review of waveform acceleration techniques based on reduced order or surrogate models that speed up parameter estimation is provided in Setyawati, Pürrer, and Ohme (2020). Vinciguerra, Veitch, and Mandel (2017) exploit the chirping behavior of compact binary inspirals to sample sparsely for portions where the full frequency resolution is not required, Zackay, Dai, and Venumadhav (2018) and N. Cornish (2013) use relative binning and the heterodyning principle, respectively, for fast likelihood evaluation. Rapid parameter inference methods using grid techniques are also being developed (Lange, O'Shaughnessy, & Rizzo, 2018; Pankow, Brady, Ochsner, & O'Shaughnessy, 2015).

To speed up parameter estimation, deep learning approaches, particularly variational autoencoders, and convolutional neural networks, have recently been explored (George & Huerta, 2018; Gabbard, Messenger, Heng, Tonolini, & Murray-Smith, 2019; Shen, Huerta, Zhao, Jennings, & Sharma, 2019; Chua & Vallisneri, 2020; S. R. Green,

Simpson, & Gair, 2020). Deep learning approaches train neural networks to learn the posterior through stochastic gradient descent to optimize a loss function. The training samples require only sampling from the prior and the likelihood which is fast. It also has the advantage that training can be performed offline and the estimation of parameters from an observed gravitational wave signal becomes almost instantaneous. These methods are still in their infancy and need to be further developed to be able to handle the full parameter space of binary inspirals and longer duration waveforms from multiple detectors. They hold great promise for low-latency parameter estimation and a fast electromagnetic follow-up.

With a whole catalog of compact black hole mergers from the first, second, and soon third LIGO – Virgo observations runs, it has now become feasible to infer population properties such as their merger rates, mass spectrum, and spin distribution, as for instance in Stevenson, Berry, and Mandel (2017); Fishbach, Holz, and Farr (2018); Wysocki, Lange, and O'Shaughnessy (2019); Chase et al. (2020); Callister, Fishbach, Holz, and Farr (2020); B. Abbott et al. (2019c); R. Smith, Talbot, Vivanco, and Thrane (2020).

4 | UNMODELED BURST SIGNALS

There are various potential origins of unmodeled short-duration *burst* signals with no known closed form, such as pulsar glitches, core-collapse supernovae, gamma-ray burst engines, and unanticipated sources. Amongst these, gravitational waves from core-collapse supernovae (CCSNe) are probably the most promising for observation (Gossan et al., 2015).

To date, gravitational waves from CCSNe have not been directly observed by the network of terrestrial detectors, Advanced LIGO and Advanced Virgo (B. Abbott et al., 2020e). However, a new era in multimessenger astronomy began with the observation of gravitational waves from a binary neutron star inspiral (GW170817) with associated counterparts observed across the electromagnetic spectrum (B. Abbott, Abbott, Abbott, Acernese, et al., 2017c). Much like GW170817, CCSNe are an important source of multimessenger astronomy as they will have associated electromagnetic, as well as neutrino counterparts (Bionta et al., 1987; Hirata et al., 1987). The gravitational wave signal from a CCSN will be of order 1 s or less. LIGO and Virgo regularly search for gravitational-wave signals from CCSN (B. Abbott, Abbott, Abraham, Acernese, et al., 2019; Abbott et al., 2020d).

Like neutrinos, gravitational waves are emitted from the core of the progenitor and carry information about the dynamics of the core-collapse and shock wave revival mechanism that leads to explosion (Kuroda, Kotake, Hayama, & Takiwaki, 2017). However, gravitational waveforms from CCSNe are analytically intractable due to the complex interplay of general relativity, particle physics, and nuclear physics, meaning template-based search methods like those employed in compact binary coalescence pipelines are currently not possible. Alternative parameter estimation routines are needed.

The first attempt to conduct parameter estimation on CCSN gravitational wave signals was by Summerscales, Burrows, Finn, and Ott (2008), who used the maximum entropy framework to deconvolve noisy data from multiple detectors to extract a CCSN gravitational wave signal. They made inferences on amplitude and phase parameters using cross-correlation between a recovered waveform and a set of competing waveforms from the Ott, Burrows, Livne, and Walder (2004) waveform catalog, where a match was defined as the model with the maximum cross-correlation to the recovered waveform.

Heng (2009) proposed simplifying the problem using principal component analysis (PCA) to reduce a supernova waveform catalog parameter space to a small number of basis vectors. Röiver et al. (2009) extended on this by creating a Metropolis-within-Gibbs sampler to reconstruct rotating core-collapse signals using principal component regression (PCR). They attempted to conduct parameter estimation by matching reconstructed signals to catalog waveforms using a χ^2 distance, but this had limited success. Edwards (2017) extended the PCR Bayesian reconstruction of CCSN signals using a birth-death reversible jump MCMC (RJMCMC) approach, allowing the number of principal components to vary, and making use of model averaging to handle the model selection problem. An example of a reconstructed CCSN waveform from the Dimmelmeier, Ott, Marek, and Janka (2008) waveform catalog can be seen in Figure 4.

Abdikamalov, Gossan, DeMaio, and Ott (2014) used matched filtering on their newly created waveform catalog to infer total angular momentum from rotating CCSN signals with errors up to 20% for rapidly rotating progenitors and 35% for slowly rotating cores. They also used nested sampling to classify precollapse differential rotation profile, with reasonable success.

FIGURE 4 Reconstructed CCSN signal from the Dimmelmeier et al. (2008) waveform catalog using a transdimensional PCR model. The true signal (solid black) and model-averaged reconstruction (dashed blue) are overlaid with the model-averaged 90% credible region (shaded pink)
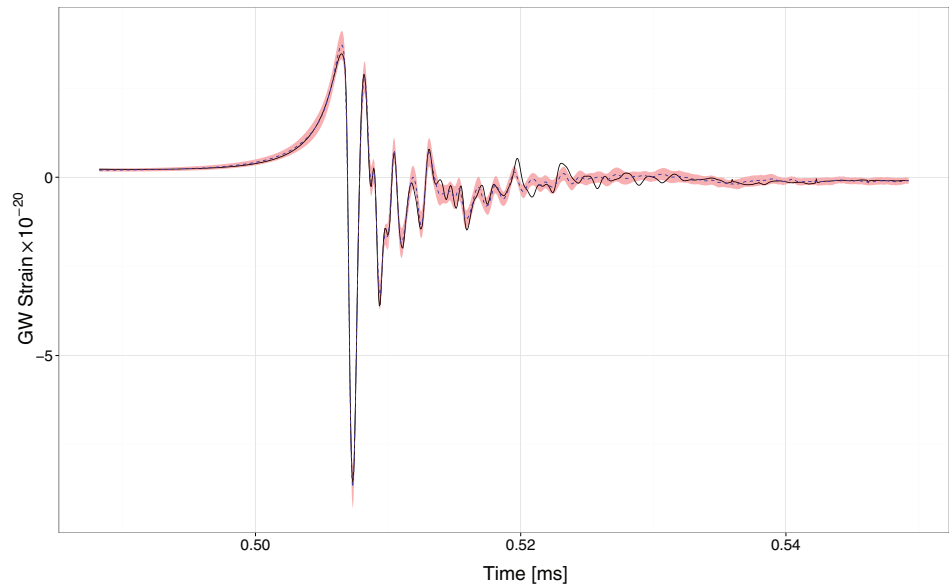
Edwards, Meyer, and Christensen (2014) demonstrated that it is possible to extract astrophysically meaningful information encoded in the posterior principal component coefficients in the Bayesian PCR model of Röiver et al. (2009). Using posterior predictive sampling, they were able to give Bayesian credible intervals for the first time on parameters such as the ratio of rotational kinetic energy to gravitational potential energy of the inner core at bounce. The authors also used supervised machine learning methods to classify the precollapse differential rotation profile.

Engels, Frey, and Ott (2014) used frequentist multivariate regression and classical hypothesis testing to analyze important astrophysical parameters from CCSNe signals. In contrast to the PCR approach of Heng (2009) and Röiver et al. (2009) to reconstruct waveforms, the authors used the method of least squares to find an encoded relationship between the PC basis functions and astrophysical parameters, identifying the most important astrophysical parameters from signals buried in simulated detector noise.

A recent extension of the PCA-based approach to parameter estimation of CCSN signals has been given by Roma, Powell, Heng, and Frey (2019). It includes features in the gravitational wave signal that are associated with g-modes and the standing accretion shock instability. Rather than computing the principal components of the simulated waveform time series, it performs a PCA on the spectrograms and test, the performance using simulated data for planned future detectors such as the Einstein Telescope and Cosmic Explorer.

The supernova explosion mechanism is not fully understood. The two most popular and well-studied supernova explosion mechanisms are the *neutrino-driven* explosion for non-rotating and slow-rotating progenitors and *magnetorotational-driven* explosion for rapidly rotating progenitors (Janka, 2012). As the gravitational wave signals from these explosion mechanisms are morphologically different, this has been an area of much focus for parameter estimation studies. This was first formulated as a classification problem by Logue, Ott, Heng, Kalmus, and Scargill (2012), where they used PCR and nested sampling (Skilling, 2006), computing Bayesian evidence to select the most likely explosion mechanism. Classifying the supernova explosion mechanism using nested sampling has been further studied by, for example, Powell, Gossan, Logue, and Heng (2016) using real detector noise, Powell, Szczepanczyk, and Heng (2017) for 3D simulations and noise transient rejection.

M. Coughlin, Christensen, Gair, Kandhasamy, and Thrane (2014), B. Abbott, Abbott, Abraham, Acernese, et al. (2019), and Banagiri et al. (2020) developed Bayesian approaches for estimating the parameters of transient signals based on the time-frequency maps. Lynch, Vitale, Essick, Katsavounidis, and Robinet (2017) explored an information-theoretic approach to the burst detection problem. Both used nested sampling as part of an algorithm for detecting short-duration gravitational-wave bursts. The method of M. Coughlin et al. (2014) can also be used for long-duration gravitational wave transients, possibly lasting up to thousands of seconds.

One of the most popular and sophisticated methods for constructing unmodeled bursts is the BayesWave algorithm by N. J. Cornish and Littenberg (2015). BayesWave uses Morlet–Gabor continuous wavelets to construct bursts and glitches. Under the reversible jump MCMC framework (P. Green, 1995), they treat the number of wavelets as variable.

Parameter estimation routines for unmodeled bursts with BayesWave are currently being developed, with reasonable success at sky localization (Becsy et al., 2017).

In line with the current trend in gravitational-wave data analysis, deep learning methods are being explored. Similar to the deep learning methods for glitch characterization (George, Shen, & Huerta, 2018; Zevin et al., 2017), convolutional neural networks have started populating the CCSN parameter estimation literature (see, for example, Astone et al. (2018); Heng and Messenger (2019); Iess, Cuoco, Morawski, and Powell (2020)) due to their success in image classification problems. However, deep learning approaches for CCSNe are still in their infancy and have not developed beyond classification problems.

5 | CONTINUOUS SIGNALS

Various sources, such as binary systems far from coalescence or non-axisymmetric rotating neutron stars will emit continuous gravitational waves. Gravitational-wave signals from rapidly rotating biaxial or triaxial neutron stars, so-called pulsars, have been searched for but have not yet been observed by Advanced LIGO—Advanced Virgo (B. Abbott et al., 2017a, 2017b, 2019e, 2019g, 2019). Pulsars emit gravitational waves that will likely be seen on Earth as weak continuous signals and are promising candidates for future observations runs. These quasi-periodic signals are of long duration with near-constant amplitude and frequency. The gravitational wave signal from such an object is at twice its rotation frequency $f_s = 2f_r$. Identification of a periodic gravitational wave signal is challenging because of the weakness of the signal. But radio observations can provide information about the sky location, rotation frequency, and spin-down rate of known pulsars, thus allowing a targeted search in a very narrow spectral window (Dupuis & Woan, 2005). The observed signal, described by

$$h(t) = F_+(t, \psi) h_0 \frac{1}{2} (1 + \cos^2 \iota) \cos \phi(t) + F_\times(t, \psi) h_0 \cos \iota \sin \phi(t),$$

depends on several unknown parameters: the overall amplitude of the gravitational wave signal h_0, the polarization angle ψ, the angle ι between spin axis of the pulsar and the line of sight, and the phase evolution $\phi(t)$. The response of the detector to the two polarization is given by $F_+(t, \psi)$ and $F_\times(t, \psi)$. The sky position parameters α and δ are fixed. A simple slowdown model provides the rotational phase evolution of the signal via a short Taylor series expansion

$$\phi(t) = \phi_0 + 2\pi \left[f_s(T - T_0) + \frac{1}{2}\dot{f}_s(T - T_0)^2 + \frac{1}{6}\ddot{f}_s(T - T_0)^3 + \cdots \right]$$

where $T = t + \delta t = t + \frac{\vec{r} \cdot \vec{n}}{c} + \Delta T$ is the time of arrival of the signal at the solar system barycenter, ϕ_0 is the phase of the signal at a fiducial time T_0, $\vec{r}$ is the position of the detector with regard to the solar system barycenter, $\vec{n}$ is a unit vector in the direction of the pulsar, c is the speed of light, and ΔT contains the relativistic corrections to the arrival time (Christensen, Dupuis, Woan, & Meyer, 2004). For most pulsars, the time derivative $\dot{f}_s$ is very small and $\ddot{f}_s$ is often swamped by timing noise. If f_s and $\dot{f}_s$ are known from radio observations, the signal can be heterodyned by multiplying the data by $\exp[-i\phi(t)]$, low-pass filtered and resampled, yielding a simple model with four unknown parameters h_0, ψ, ι, ϕ_0. If there is an uncertainty in the frequency and frequency derivative then we have two additional parameters, the differences between the signal and heterodyne frequency and frequency derivatives (Christensen, Dupuis, et al., 2004; Dupuis & Woan, 2005; Umstätter et al., 2004). Early MCMC techniques for sampling from the posterior distribution used a combination of reparametrizaton, delayed rejection, and simulated annealing Umstätter et al. (2004). Nested sampling code for parameter estimation and model selection in targeted searches for continuous gravitational wave signals from pulsars has been developed by Pitkin, Gill, Veitch, Macdonald, and Woan (2012) and implemented in the `LALInference` software and is described in detail in Pitkin, Isi, Veitch, and Woan (2017). Ashton and Prix (2018) introduce a method for the hierarchical follow-up of continuous gravitational wave candidates by leveraging MCMC optimization of the F-statistic using the affine-invariant ensemble sampler (Foreman-Mackey et al., 2013).

Dreissigacker, Sharma, Messenger, Zhao, and Prix (2019) highlighted that deep learning, particularly convolutional neural networks, can be used to directly search for continuous waves. Though the results were much faster than matched filtering, the approach needs to be further developed to be more competitive (in terms of the detection probability) with existing methods.

6 | STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

The combination of all gravitational wave signals that cannot be individually resolved will make up what is called the stochastic gravitational-wave background (SGWB; Christensen, 2019; Romano & Cornish, 2017). Analogous to the cosmic microwave background, the physical processes in the early evolution of the Universe created the *cosmological* SGWB. The superposition of many unresolved signals from many independent sources such as the galactic population of white dwarf binaries, compact binary mergers, supernovae, pulsars, magnetars, and cosmic strings make up the *astrophysical* SGWB. As electromagnetic waves cannot provide information about astrophysical sources and processes any earlier than 400,000 years after the Big Bang—the time of last scattering—detection and estimation of the SGWB is extremely important to probe the early Universe. Unlike transient gravitational-wave signals that come from a certain location in the sky, the SGWB signal will come from all directions and may or may not be isotropic and uniformly distributed across the sky (B. Abbott et al., 2017e, Abbott, Bloemen, Groot, Nelemans, & Schmidt, 2019). By and large, the SGWB is a stochastic signal and will be another source of noise in a single detector, often modeled as stationary Gaussian with mean zero and positive definite covariance matrix or spectral density to be estimated. So the fundamental problem is to distinguish the SGWB "noise" from instrumental noise (M. Adams & Cornish, 2010). When there are several detectors such as in the network of Advanced LIGO – Advanced Virgo, cross-correlation methods can be employed, for example, with observations at two detectors

$$d^{(k)}(t) = h(t) + n^{(k)}(t), \quad k = 1, 2$$

assuming independent noise components, the correlation between the observations becomes (Christensen, 1992):

$$Cov\left(d^{(1)}(t), d^{(2)}(t)\right) = Cov\left(h(t) + n^{(1)}(t), h(t) + n^{(2)}(t)\right) = Cov(h(t), h(t)).$$

Advanced LIGO and Advanced Virgo have used these correlation methods to search for the SGWB. Even though no SGWB signal has been detected to date, upper limits have been placed on the energy density of the SGWB within the frequency range of 20–1,000 Hz (B. Abbott, Abbott, Abbott, Abernathy, et al., 2017e; Abbott et al., 2019). The energy density of the SGWB is assumed to have the form

$$\Omega_{\text{GW}}(f) = \Omega_\alpha \left(\frac{f}{f_{\text{ref}}}\right)^\alpha. \tag{13}$$

$\Omega_{\text{GW}}(f)$ is the power spectral density of the SGWB divided by the *closure density* of the universe $\rho_c = 3H_0^2/(8\pi G)$, and the Hubble constant is $H_0 = 67.9$ km s^{-1} Mpc^{-1} (Ade et al., 2016). The slope of the energy spectral density α is assumed to be zero for a cosmological background, and 2/3 for a background created by binary black hole and binary neutron star mergers throughout the history of the universe. A reference frequency f_{ref} defines where the amplitude Ω_α is measured and reported. In the latest results reported by LIGO and Virgo based on the observations from their second observing run, there was no detected SGWB, and parameter estimation methods were used to generate posterior distribution functions for Ω_α and α. The Bayesian parameter estimation method used by LIGO-Virgo for the SGWB search was first presented by Mandic, Thrane, Giampanis, and Regimbau (2012). For the constraint of $\alpha = 0$, a presumed cosmological background, a 95% credible level upper limit was set at 25 Hz to be $\Omega_0 < 6.0 \times 10^{-8}$, while for a compact binary produced background with $\alpha = 2/3$ the upper limit for $f_{\text{ref}} = 25$ Hz is $\Omega_{2/3} < 4.8 \times 10^{-8}$ (B. Abbott et al., 2019i).

General relativity predicts that the polarization of gravitational waves would only have a tensor form, while alternate theories of gravity predict vector and scalar polarizations. LIGO and Virgo have searched for the presence of these alternate polarizations, and have not found a signal. Using nested sampling, a Bayes factor has been used to compare the presence of a signal to Gaussian noise. Bayes factors have been computed in searches for all three polarizations, and for the results to date for LIGO and Virgo no SGWB of any polarization is observed (Callister et al., 2017; B. Abbott et al., 2018; Abbott, Abbott, Abbott, Abraham, et al., 2019i).

The future space-based interferometer space antenna LISA will search for a SGWB in the 10^{-5} Hz to 10^{-1} Hz frequency band (Amaro-Seoane et al., 2017). LISA will yield observations from thousands of sources of which many will remain unresolved such as gravitational waves from the galactic white dwarf binaries which will form a foreground

and in addition to the instrument noise will make the estimation of the SGWB even more difficult (Barack & Cutler, 2004; M. R. Adams & Cornish, 2014; Sachdev, Regimbau, & Sathyaprakash, 2020). LISA is comprised of three coupled interferometers. A principal component-like combination of the three output signals is made to eliminate the effect of this correlated noise. This is called time delay interferometry (TDI; Tinto & Dhurandhar, 2014; Vinet, 2013). When expressed as the orthogonal modes (A, E, T), one of them, T, is insensitive to the gravitational wave signal at low frequencies; this channel can be used to understand the noise. The other two channels are orthogonal. This means that auto-correlation methods must be used to try to observe the SGWB (Romano & Cornish, 2017; T. L. Smith & Caldwell, 2019), with the null channel employed to disentangle the detector noise from the SGWB signal. New search strategies and parameter estimation methods such as for instance in Caprini et al. (2019) need to be developed that can be applied to non-Gaussian, anisotropic, circularly polarized backgrounds, and backgrounds with polarization components predicted by alternative theories to general relativity (Romano & Cornish, 2017). This is currently an active area of research and Bayesian parametric and nonparametric methods for spectral density estimation of time series as reviewed in Section 7 will be important.

7 | INTERFEROMETER NOISE

As described in Section 2, for the purpose of estimating transient signals, the power spectral density of the noise $n(t)$ is usually estimated "off-source", that is, from a separate stretch of data not containing the signal using Welch's method and is then assumed to be fixed and known. For short duration signals, it might be appropriate to assume that the noise process is stationary and that the PSD thus does not change over time, however, for longer duration signals, this is not a reasonable assumption. Similarly, the assumption of *Gaussianity* is questionable considering the large number of transient noise events, known as glitches, for example, from environmental sources, weather, or equipment faults. Residuals from parameter estimation using certain noise assumptions need to be carefully checked as for instance described in B. Abbott et al. (2020a). Even if the noise is stationary, estimating the PSD from a separate data sequence and then assuming it to be fixed for the purpose of signal parameter estimation ignores any uncertainty in the PSD estimate and can thus lead to biased signal parameter estimates (Chatziioannou et al., 2019).

For an accurate estimation of signal parameters as well as sensitive and confident signal detection (Venumadhav, Zackay, Roulet, Dai, & Zaldarriaga, 2019), it has been realized that a realistic modeling of the interferometer noise is extremely important. Such a noise model should be able to handle non-Gaussian and time-varying noise and to be included in an "on-source" method, that is, a method that estimates both signal and noise parameters simultaneously. Various approaches have been suggested in the literature that achieve some of these goals.

To be able to track changes in the PSD over a long period of time, Cuoco (2001) suggested to split the time series into smaller chunks and estimate the PSD using classical parametric spectral density estimation methods based on fitting autoregressive (AR) or autoregressive moving average models. Zackay, Venumadhav, Roulet, Dai, and Zaldarriaga (2019) deal pragmatically with alleviating the effect of non-stationary noise on signal detection by dividing the matched filtering overlaps by their locally estimated standard deviations.

For stationary Gaussian noise n with an unknown PSD, a prior model for $S(f)$ in the Whittle likelihood (4):

$$L(\mathbf{n}) \approx \frac{1}{\det(\pi T \mathbf{S})} e^{-\frac{1}{T}\tilde{n}^* \mathbf{S}^{-1} \tilde{n}} = \exp\left\{ -\sum_j \left[\frac{|\widetilde{n}(f_j)|^2}{TS(f_j)} + \log\left(\pi TS(f_j)\right) \right] \right\} \tag{14}$$

needs to be specified. Röver, Meyer, and Christensen (2011) modeled the unknown spectral density components using conjugate inverse Gamma distributions, yielding Student-t marginal distributions for the errors and enabling to accommodate outliers in the data. Similarly, T. B. Littenberg, Coughlin, Farr, and Farr (2013) and Veitch et al. (2015) incorporate uncertainty about the estimated PSD by an additional scale factor η_j for each frequency bin, that is, replacing $S(f_j)$ by $\eta_j S(f_j)$, and giving it a Normal prior with mean 1, where $S(f_j)$ is estimated beforehand using the Welch method. BayesLine, a flexible Bayesian spectral density estimation method for Gaussian stationary noise that has been widely applied for gravitational wave data analysis was developed by T. B. Littenberg and Cornish (2015). BayesLine models the smooth part of the PSD by a linear combination of cubic splines where the number as well as the knots of the basis splines are unknown parameters. The spectral lines in the PSD are modeled using a sum of Lorentzians where the

number, location, and line width are unknown parameters. A reversible jump MCMC algorithm is used to sample from the posterior distribution. This off-source algorithm is then extended in N. J. Cornish and Littenberg (2015) to an on-source method, known as BayesWave, by simultaneously fitting a gravitational wave burst signal and potential glitches, both modeled as a sum of Morlet-Gabor continuous wavelets, see also Section 4. Whereas BayesWave can reconstruct the gravitational wave signal as demonstrated for instance in Figure 3 it does not provide estimates of the physically meaningful waveform parameters. Biscoveanu, Vitale, and Davies (2020) combined the BayesLine model for the PSD with the physical CBC waveform models to simultaneously estimate the signal parameters and the PSD. By marginalizing over the PSD, the marginal posterior distributions of the signal parameters take the full uncertainty of the PSD estimates into account.

Instead of simultaneously estimating the spectral density and waveform parameters Talbot and Thrane (2020) developed a variant of Welch method by taking the median instead of the average of periodograms over neighboring segments. The likelihood is derived after marginalization over the uncertainty in the median PSD estimate. The analysis is shown to be robust with respect to large outliers.

Approaches based on a parametric model for the spectral density, such as those based on fitting autoregressive moving average (ARMA) models, can be very efficient when the parametric model is correctly specified but can lead to biased results under model misspecification. Nonparametric models, on the other hand, have much wider applicability and robustness as they do not rely on finite-dimensional distributional assumptions. A nonparametric on-source approach, treating the spectral density function $S(f)$ as an infinite-dimensional parameter and modeling this nonparametrically using a Bernstein–Dirichlet process prior (Choudhuri, Ghosal, & Roy, 2004) has been developed by Edwards, Meyer, and Christensen (2015) and used to simultaneously fit rotating core collapse supernova gravitational wave burst signals embedded in simulated Advanced LIGO—Advanced Virgo noise. To improve the approximation of spectral lines, this nonparametric method was modified to use B-splines instead of Bernstein polynomials. By putting a Dirichlet process prior on the knot differences, the B-spline-Dirichlet process prior was shown to be able to accurately pick up sharp peaks and spectral lines in the data from the LIGO S6 science run (Edwards, Meyer, & Christensen, 2019). The method is implemented in the R package `bsplinePsd` (Edwards, Meyer, & Christensen, 2018). It has also been used as on on-source model for simultaneously estimating parameters of a non-chirping galactic white dwarf binary signal in simulated LISA data (Edwards et al., 2020). An MCMC algorithm combined with parallel tempering was used for posterior computation. A recent modification that reduces the computational complexity while keeping the good approximation and coverage properties of the B-splines by using P-splines, that is, B-splines but with fixed knots and a smoothness penalty on the coefficients, is given in Maturana-Russel and Meyer (2019) and implemented in the R package `psplinePsd` (Maturana-Russel & Meyer, 2020). By taking advantage of a well-fitting parametric autoregressive model, Kirch et al. (2019) can improve on the Whittle likelihood approximation using a nonparametric correction of a parametric working model and prove posterior consistency. Using the Bernstein–Dirichlet process prior for the spectral density, they demonstrate improved performance using the same S6
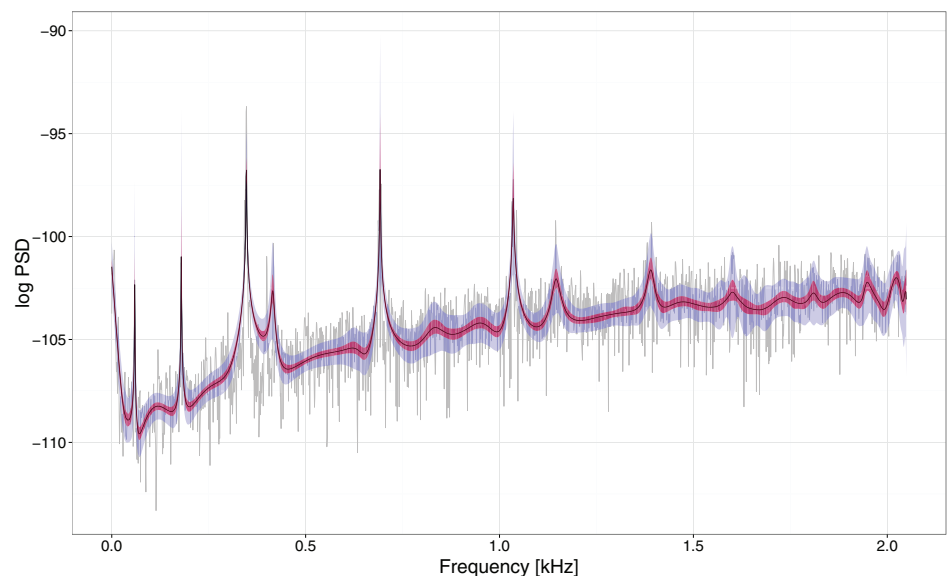

FIGURE 5 Estimated log spectral density for a 1 s segment of Advanced LIGO S6 data. The posterior median log spectral density estimate using the corrected likelihood with an AR(35) working model (solid black), pointwise 90% credible region (shaded red), and uniform 90% credible region (shaded violet) are overlaid with the log periodogram (gray; Kirch et al., 2019)

LIGO noise data as Edwards et al. (2019). A spectral density estimate based on the corrected likelihood and an autoregressive working model is shown in Figure 5. Sampling is based on adaptive Metropolis-Hastings steps within the Gibbs sampler, implemented in the R package `beyondWhittle` (Meier, Kirch, Edwards, Meyer, & Christensen, 2018). These nonparametric approaches to spectral density estimation can be used to simultaneously estimate the waveform parameters in a Gibbs step, as demonstrated for instance in Edwards et al. (2015).

8 | CONCLUSION

Bayesian methods for parameter estimation of gravitational wave data have proven to be essential and effective for analyzing the events detected by Advanced LIGO and Advanced Virgo (B. Abbott, Abbott, Abbott, Abraham, et al., 2019d), and will play an equally important role for the future space-based detector LISA (Amaro-Seoane et al., 2017). The two mainstays of posterior computational techniques that are routinely used and implemented in LALInference (Veitch et al., 2015) are parallel tempering MCMC and nested sampling. The main problems with either algorithm when exploring the high-dimensional parameter space are potentially getting stuck in local maxima and slow mixing. These are due to the multimodality of the posterior distribution and the inherent sequential Markov chain steps of both algorithms that yield slow convergence when there are high correlations between the parameters. Any convergence acceleration methods that can yield better mixing, for example, via adaptive MCMC methods (Barber et al., 2011) and enhancements of nested sampling (Brewer & Foreman-Mackey, 2018; Feroz & Skilling, 2013) will be critical. Reduced order models and surrogate waveform models (R. Smith et al., 2016; Setyawati et al., 2020) have and will continue to play a role in the acceleration of posterior computations. Furthermore, the development of nonparametric approaches will allow to make the inference more robust with respect to certain assumptions.

An alternative inferential framework which holds great promise for the future of gravitational wave analysis and has seen an enormous increase in research activity is *deep learning*. This is a machine learning technique that is extremely scalable and can learn from raw data by using deep hierarchical layers of neural networks combined with optimization techniques based on back-propagation and gradient descent (Goodfellow, 2016). It also allows to take account of deviations from the usual assumption of stationary Gaussian noise, as it can be trained on signals embedded in non-Gaussian and non-stationary noise. Whereas deep learning has so far been mainly employed for detection and classification problems, see for example, Gabbard, Williams, Hayes, and Messenger (2018), George et al. (2018), Gebhard, Kilbertus, Harry, and Schölkopf (2019), S. Coughlin et al. (2019), Corizzo, Ceci, Zdravevski, and Japkowicz (2020), Beheshtipour and Papa (2020), and Cuoco et al. (2020), recent research has a focus on the parameter estimation problem (Chua & Vallisneri, 2020; Fan, Li, Li, Zhong, & Cao, 2019; George & Huerta, 2018). Its main advantage is the fact that the time-consuming training of the neural nets can be performed off-line and then potentially render the parameter estimates of an observed gravitational event in an instant.

In their third observing run, O3, Advanced LIGO and Advanced Virgo reported potential detections at a cadence of about one per week (LIGO Scientific Collaboration, Virgo Collaboration, 2020a). This rate will increase in the upcoming observing runs as the sensitivity of the detectors improves (B. Abbott et al., 2018). Bayesian parameter estimation will continue play a critically important role in the description of the physical systems that are producing the gravitational wave events.

AUTHOR CONTRIBUTIONS

Renate Meyer: Conceptualization; funding acquisition; methodology; project administration; supervision; visualization; writing-original draft; writing-review and editing. **Matthew Edwards:** Methodology; software; writing-original draft; writing-review and editing. **Patricio Maturana-Russel:** Methodology; software; writing-original draft; writing-review and editing. **Nelson Christensen:** Funding acquisition; supervision; writing-original draft; writing-review and editing.

CONFLICT OF INTEREST

The authors have declared no conflicts of interest for this article.

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