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Momentum distributions for the quantum δ-kicked rotor with decoherence

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Abstract. We discuss the theoretical description and numerical simulation of the effect of noise and environmental coupling on the momentum distribution of the quantum δ-kicked rotor in recent atom optics experiments. We investigate the transition between exponentially localized and more nearly Gaussian momentum distributions and compare our simulations with existing experimental results.

Keywords: Quantum chaos, decoherence, laser-cooled atoms

1. Introduction

There has been a renewed interest in recent years in the correspondence between quantum and classical mechanics for systems which are classically chaotic. It is frequently the case that the quantum dynamics of such systems are dramatically different from their classical counterparts. In the case of the δ-kicked rotor (DKR) or standard map, a canonical example of a classically chaotic system, the classical diffusion is completely suppressed by the quantum dynamics, and momentum distributions become exponentially localized [1]. The quantum DKR (Q-DKR) has in recent years become accessible in atom optics experiments [2], which makes this system a useful testing ground for our understanding of the transition between quantum and classical dynamics.

It is thought that decoherence [3] plays a significant role in the quantum-to-classical transition in classically chaotic systems [4, 5]. Any real quantum system is in fact an open system coupled to some extent to the surrounding environment and such couplings typically destroy the quantum coherences on which phenomena such as dynamical localization rely. Decoherence in the Q-DKR has been addressed in various theoretical studies [6,7]. Coupling to the environment, which can for example take the form of noise in the potential, does typically lead to a continual increase in the energy of the quantum system and much previous work has focused on calculating the associated diffusion rate. However, it is also of interest to investigate the effect of decoherence on the exponentially localized momentum distributions and the transition—if any—to the Gaussian momentum distribution predicted by the classical standard map. Recent experiments utilizing ultracold caesium atoms have investigated the dynamics of the Q-DKR with various forms of decoherence [8–12]. However, there is still much room for theoretical analysis of the specific physical systems realized by these experiments—in this work we mainly consider the question of momentum distributions but our methods would be suitable for more general theoretical treatments of the experiments.

In our previous publication [8] we noted that for the spontaneous emission rates used in our experiment, the momentum distribution shapes remained exponential; examples of these momentum distributions were published in [9, 11]. The Austin group have observed momentum distributions that are clearly not exponentially localized [10]. It is important to note, however, that the two results are not necessarily inconsistent. The Austin group's spontaneous emission rate per kick (13%) was greatly in excess of our highest rate of 4.6%. At the relatively low spontaneous emission rates used in our experiment the momentum distribution remains essentially exponential, at least for the number of kicks investigated in our experiment. Delocalization is reflected by a growing momentum variance rather than by a transition from exponential to Gaussian (or some other) lineshapes, at least for early times. A similar behaviour has been found in the theoretical investigation of a phase-modulated potential [13]. However, for sufficiently strong decoherence, there is also a transition in the momentum lineshape from exponentially localized to essentially Gaussian within the period of the experiment. This transition, observed in the Austin group's experiments, is also clearly visible in our numerical simulations. Note that since the effect of the spontaneous emission is essentially perturbative it might be expected that the transition in the shape of the distribution will eventually occur even for very small spontaneous emission probability given a sufficiently large number of kicks. In this work we focus on a fixed number of kicks and investigate the transition as a function of the spontaneous emission rate since very large numbers of kicks are not achievable in the experiments.

2. The atom optics realization of the DKR

We consider an atom (transition frequency ω_0) suspended in a standing wave of near-resonant light (frequency ω_l or wavenumber k_l). Both the Auckland [8] and Austin [10] groups studied decoherence in the DKR by using laser-cooled caesium atoms and so we will base our model of the atom optics realization of the DKR on the caesium atom. In this section we will neglect spontaneous emission and just consider the coherent dynamics of the atom; the effects of decoherence will be discussed in detail in the next section. Under the assumption of large detuning compared with the Rabi frequency, the resulting Hamiltonian governing the coherent time evolution is

$$H = \frac{p^2}{2m} - \frac{\hbar\Omega_{\rm eff}}{8}\cos(2k_l x)\sum_{n=1}^{N} f(t-nT) \qquad (1)$$

where $\Omega_{\rm eff} = \Omega^2(s_{45}/\delta_{45} + s_{44}/\delta_{44} + s_{43}/\delta_{43})$ and $\Omega/2$ is the resonant Rabi frequency corresponding to a single beam. The terms in brackets take account of the differing dipole transitions between the relevant hyperfine levels in caesium ($F = 4,\ F' \to 5, 4, 3$). The δ_{4j} are the corresponding detunings and, assuming equal populations of the Zeeman sublevels, the numerical values for the s_{4j} are $s_{45} = \frac{11}{27}$, $s_{44} = \frac{7}{36}$, $s_{43} = \frac{7}{108}$. The function $f(t)$ represents the shape of the kicks, which in this work is close to rectangular: $f(t) = 1$ for $0 < t < \tau_p$ and zero otherwise. In the limit where $\tau_p \to 0$ we recover the DKR. The pulses repeat with period T. It is convenient to convert to dimensionless units and the resulting dimensionless Hamiltonian for the kicked rotor is

$$H' = \frac{\rho^2}{2} - k\cos(\phi)\sum_{n=1}^{N} f(t-n) \qquad (2)$$

where $\phi = 2k_l x$, $\rho = 2k_l T p/m$, $t' = t/T$ and $H' = (4k_l^2T^2/m)H$; the primes are subsequently dropped. The Hamiltonian only couples momenta separated by $2\hbar k_l$ so it is sometimes useful to consider the integer momentum $n = p/2\hbar k_l$. The classical stochasticity parameter is $\kappa = \Omega_{\rm eff}\omega_R T\tau_p$, with $\omega_R = \hbar k_l^2/2m$ the recoil frequency and $k = \kappa T/\tau_p$. The quantum features of the DKR enter through the commutation relation $[\phi, \rho] = {\rm i}\,\bar{k}$, where $\bar{k} = 8\omega_R T$.

In both experiments caesium atoms are initially cooled in a magneto-optic trap (MOT) to a temperature of $\sim$10 μK. Then the time-dependent periodic potential is generated by pulsing a laser beam on and off. Finally the momentum distribution after the last laser pulse is measured using a time-of-flight technique with a 'freezing molasses' [2]. In our experiments [8, 9] the probability of a spontaneous emission taking place during one of the laser pulses is increased by tuning the laser more closely to the atomic resonance; the depth of the potential is kept constant by shortening the length of the laser pulses. In the Texas experiments spontaneous emission is introduced by leaving on the molasses laser beams which are responsible for the cooling of the atoms in the MOT [10]. In this latter experiment the effect of noise is also investigated by randomly changing the power of the laser used in each successive laser pulse, which has the effect of randomly modifying the depth of the potentials seen by the atoms.

3. Theoretical considerations

We performed numerical simulations of the Q-DKR modified in order to include the effects of noise and spontaneous emission. Two important considerations in designing our simulations were correctly modelling the experimental initial state and taking advantage of the spatial periodicity of all three models (unitary Q-DKR, Q-DKR with spontaneous emission and Q-DKR with noise). The periodicity in each case is that of the intensity of the standing wave or half the wavelength of the light. In the experiments, the atomic cloud after the laser cooling has a broad distribution in position compared with the wavelength of the light and a nearly Gaussian distribution in momentum. Thus the state of atoms in the cloud may be approximated, at least near the centre of the cloud, by a mixed state with a uniform, and thus periodic, position distribution and the appropriate Gaussian momentum distribution. We can normalize the state over one period or half the wavelength of the light. The resulting density matrix can be written as an incoherent sum of momentum eigenstates and in our simulations we randomly select an initial (pure) state for each trajectory from this mixture of momentum eigenstates and average over these initial states. These are essentially the same conclusions about the appropriate initial state as reached in [14]. The translational symmetry of the models means that the state will always have the Bloch form $\exp({\rm i}q\phi)u_q(\phi)$, where u_q has a period of $\lambda/2$ and q is the of the quasi-momentum. This Bloch form will be preserved by the dynamics and indeed all the dynamics except for spontaneous emission events will preserve the quasi-momentum on each trajectory. The periodic wavefunction u_q may be propagated on a discrete basis set appropriate for a periodic phase space where the appropriate kinetic energy operator is a function of the quasi-momentum. A fast Fourier transform (FFT) algorithm for the propagation of the state vector then naturally enforces the periodicity of u_q.

In the case of spontaneous emission, the motional state of the atom is coupled to an environment that is made up of the internal electronic states of the atom and the electromagnetic vacuum into which the atom spontaneously emits. Typically though, the rates which characterize those internal dynamics are larger than those that determine the dynamics of the motional state. This difference of timescales gives rise to the familiar picture of atomic motion in an optical potential, characterized in the semiclassical limit by a conservative force, a friction force and momentum diffusion due to spontaneous emission and fluctuations in the dipole force [17, 18]. In this work we are interested in the full quantum mechanical evolution of the motional state, which will typically require consideration of some master equation for the atomic motion in this same limit where the motional dynamics are slower than the internal dynamics. In order to model the system evolution with spontaneous emission in our experiment we have performed Monte Carlo wavefunction simulations of the master equation for the motion of a two-level atom in a far-detuned optical standing wave [19]:

$$\begin{aligned}\dot{\rho} = & -\mathrm{i}[H,\rho] - \frac{\eta}{\alpha}\sum_{n=1}^{N} f(t-n)\{\cos^2(\phi/2),\rho\} \\ & + 2\frac{\eta}{\alpha}\sum_{n=1}^{N} f(t-n)\int_{-1}^{1}\mathrm{d}u\, N(u)\mathrm{e}^{\mathrm{i}u\phi/2} \\ & \times \cos(\phi/2)\rho\cos(\phi/2)\mathrm{e}^{-\mathrm{i}u\phi/2},\end{aligned} \tag{3}$$

for which $N(u)$ is the (suitably normalized) distribution of spontaneous emission recoil momenta projected onto the standing wave axis and $\alpha = \tau_p/T$. The probability of a single spontaneous emission during one kick is labelled η. This master equation is valid for large detunings for which the saturation parameter $\Omega_{\mathrm{eff}}/\delta \ll 1$; outside this regime the force and momentum diffusion will be modified from the values predicted by this master equation and there may be a significant velocity-dependent cooling or heating force. In our experiment, $\Omega_{\mathrm{eff}}/\delta$ takes on a range of values 0.006–0.25. The last term describes, fully quantum mechanically, the momentum diffusion due to spontaneous emission during the kicks. Spontaneous emission is most likely where the standing wave is most intense, hence the cosinusoidal variation of Lindblad operators. The first term describes the motion of the atom in the potential provided by the light field. Note that, because we are interested only in motion along the standing wave, the recoil kick experienced by the atom due to a spontaneous emission may take any value less than $\hbar k_l$. The value of η can be modified relative to k by changing the detuning of the laser frequency thus making spontaneous emission a more or less significant factor in the dynamics.

It is worthwhile comparing the sources of momentum diffusion in this master equation with the semiclassical theory. Using standard techniques [15] it is straightforward to derive a differential equation for the momentum variance from the master equation (3)

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\langle(\Delta p)^2\rangle = & \frac{\eta}{2\alpha}\,\bar{k}^2\sum_{n=1}^{N} f(t-n)(\beta\langle\cos^2(\phi/2)\rangle \\ & + \langle\sin^2(\phi/2)\rangle) + \cdots,\end{aligned} \tag{4}$$

where $\beta = \int u^2 N(u)\,\mathrm{d}u$ is a constant of order unity and where terms resulting from the Hamiltonian have been suppressed. These two terms correspond to the two contributions to the semiclassical momentum diffusion coefficient in this low-saturation limit [17] and describe the increase in atomic energy due to the coupling to the environment. The suppressed terms result from the unitary dynamics. The first contribution is associated with the recoil kicks due to spontaneous emission and varies as the standing wave intensity. The second contribution, which varies as $\sin^2(\phi/2)$, is sometimes termed the reactive diffusion and results from the interaction of the field gradient with the atomic dipole fluctuations [17].

Our simulations of this master equation are based on the work of Marte *et al* [16] and take advantage of the fact that the master equation for spontaneous emission is still form invariant under translations of half the optical wavelength. The master equation is simulated through the well-known technique of quantum trajectories. Between spontaneous emission events the state-vector evolves according to a non-Hermitian Hamiltonian. When the norm falls below a randomly chosen value, a spontaneous emission is deemed to have occurred and a collapse operator is applied to the state, which is then normalized, and the process starts again. It turns out that on each trajectory the quasi-momentum is preserved between spontaneous emission events. After each spontaneous emission a new quasi-momentum results and this leads to an effective shift in the kinetic energy operator for the subsequent evolution. In contrast to our earlier simulations, the intensity dependence of the spontaneous emission probability and the possibility of there being more than one spontaneous emission per kick is taken into account.

The experiments are performed with caesium atoms and, particularly for moderate detunings, the multi-level nature of the transition will lead to departures from the simplified master equation we have considered here. In particular, transitions between the different Zeeman ground states may occur on optical pumping timescales and lead to Sysiphus cooling for example. These optical pumping timescales may in fact be sufficiently slow that the ground states will have to be included in the master equation explicitly, leading to an effectively non-Markovian noise source where the direction of spontaneous emission events is correlated with the history of the occupation of the internal atomic ground states. In the Austin experiment, the spontaneous emission probability is obtained by turning on the molasses beam, which is significantly closer to the atomic resonance than the kicking beam and for which such transitions are certainly important. This is evidenced by the cooling effect of the beams observed when the system is not periodically kicked [10].

Another possible difference between the Austin [10] and Auckland [8, 9] experiments is that spontaneous emission occurs in the Austin experiment mainly during the free evolution and in our experiment essentially only during the kicks. To model this situation we used a master equation for which the terms referring to the spontaneous emission act continuously and are not pulsed with the kicking laser. The contributions are rescaled in order to give the same probability of spontaneous emission per kick, resulting in the following master equation:

$$\begin{aligned}\dot{\rho} = & -\mathrm{i}[H,\rho] - \eta\{\cos(\phi/2)^2,\rho\} \\ & + 2\eta\int_{-1}^{1}\mathrm{d}u\, N(u)\mathrm{e}^{\mathrm{i}u\phi/2}\cos(\phi/2)\rho \\ & \times \cos(\phi/2)\mathrm{e}^{-\mathrm{i}u\phi/2}.\end{aligned} \tag{5}$$

It should be noted that this disregards the effect of both the friction force and the light pressure force due to the molasses beams; however, such a treatment appears to be sufficient to reproduce the essential features of the momentum distribution observed in the experiment. The cooling dynamics and the light pressure force could be included in a multilevel atom treatment employing the same theoretical and numerical method, as in [16]. For the parameters of the Austin experiment it turns out to make little difference whether the spontaneous emission is during the kicks or the free evolution. The essential difference between the momentum distributions observed by the two groups is the extent of the spontaneous emission—a maximum of 13% per kick for the Austin experiment and 4.6% for the Auckland experiment.

In our experiment the spontaneous emission probabilities are altered by changing the detuning of the kicking beam and the length of the kicks. The experiments with the smallest spontaneous emission probability have the longest pulse lengths and as a result for the 0.76% case, as noted in [8], there is a KAM (Kolmogorov–Arnol'd–Moser) boundary at $n \simeq 87$. By comparing our simulations for this value of α and for the smaller value appropriate for the 4.6% case we find that this boundary has the effect of weakly suppressing the energy growth for this lowest value of spontaneous emission. On the other hand the Austin experiment is carried out at a constant value of α since the kicking beam itself has no role in producing the spontaneous emission.

Unlike the phase space of a rotor which is truly periodic, the atom optics experiments in fact realize a kicked particle for which the discrete bands are supplemented by a continuous range of quasi-momenta. This can lead to qualitatively different behaviours of the two systems; see for example [14]. Cohen has found that certain forms of noise when applied to the kicked particle lead to non-perturbative energy growth [7, 20], which does not occur when the same noise is introduced to the Q-DKR proper. In Cohen's work a random linear potential (which certainly breaks the translational symmetry of the model) is applied along with each kick and the resulting energy growth scales as the cube root of the noise power (in the case of spontaneous emission η may be identified as characterizing the noise power). As noted in [8] there is no evidence of such a non-perturbative dependence on the spontaneous emission probability in the experiments or in the simulations employed there. This is true also of our new more rigorous numerical calculations. In [20] it is suggested that this non-perturbative energy increase occurs as a result of arbitrarily small changes of momentum occurring as a result of the noise source. However, such small momentum changes are certainly present in our model as a result of spontaneous emission since the projection of the wave vector of the spontaneously emitted photons onto the standing wave axis means that any change of momentum smaller than $\hbar k_l$ is possible as a result of a spontaneous emission event. Note that in our simulations the quasi-momentum is treated rigorously as a continuous degree of freedom and thus any non-perturbative behaviour should not be hidden by discretization of the phase space. This would appear to require a modification of the reasons suggested in [20] for the origin of this non-perturbative behaviour. Perhaps the fact that the momentum kicks due to spontaneous emission have a typical size of the order of $\hbar k_1$ is sufficient to prevent the non-perturbative behaviour even though spontaneous emission results in some arbitrarily small momentum kicks. On the other hand the absence of this effect could well be related to the fact that the translational symmetry of the model is preserved even in the presence of spontaneous emission, an explanation more in line with that given in [7]. If this is the case, the observation of this non-perturbative dependence of the energy on the noise power would require a noise source, which introduces at least one new spatial periodicity to the problem. Such an experiment would also require strategies for numerical simulation rather different from the ones employed here.

In the noise experiments performed by the Austin group, the kick strengths k_n for each kick are randomly chosen from a uniform distribution about the mean value k. The range of possible values of k_n is expressed as a percentage of k and this determines the strength of the resulting noise. In simulating these experiments, different random kick sequences were determined and the resulting sequence of Hamiltonians applied to initial pure states drawn from the thermal ensemble.

4. Results

We previously reported on our experimental measurements of the growth of the atoms' kinetic energy with time [8, 9]. This was done for $\kappa = 12.5$, $\bar{k} = 2.1$ and spontaneous emission rates of $\eta = 0.76$, 2.3 and 4.6% per kick. A picture of the characteristic exponentially shaped momentum distribution was also presented for the $\eta = 0.76\%$ example [9] and for all three levels of spontaneous emission in [11]. The spontaneous emission introduces decoherence to the Q-DKR which results in quantum diffusion [8], or momentum diffusion after the quantum break time. For the levels of spontaneous emission and the number of kicks in our experiments the lineshapes remain essentially exponential in shape. So delocalization emerges through an increase in the momentum variance and not a change in the character of the lineshape, at least over 100 kicks. Experimental limitations prevent the measurement of lineshapes for kick numbers in excess of $\sim$100. However, for stronger spontaneous emission, such as in the Austin experiment, the shape of the momentum distribution itself is modified.

Dynamical localization is characterized by a constant limiting energy (absence of classical diffusion) and exponential momentum distributions. These results indicate that localization is destroyed in two steps in the presence of spontaneous emission. At low noise levels the energy continues to grow with each kick while the momentum distribution remains exponential. This destroys localization in the sense that the energy now increases without bound. The second step of this process is the change in momentum distribution from exponential to Gaussian which takes place at higher noise levels or later times. To highlight this transition in the momentum distribution we present, in figure 1, theoretical momentum distributions for spontaneous emission rates of 0.76, 2.3 and 13% after 100 kicks. The values of κ and $\bar{k}$ are those of our experiment and the spontaneous emission takes place during the kicks. For clarity, this graph is plotted at constant kick length α for the different values of η. For our experimental values of α,

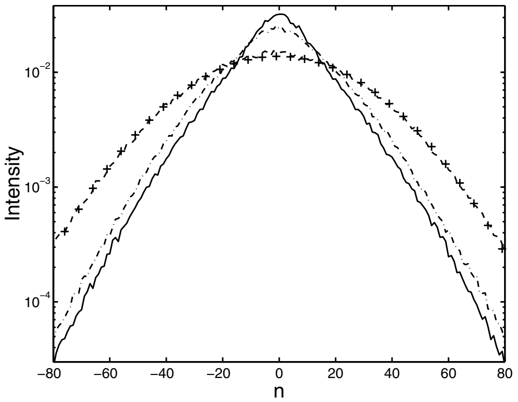

Figure 1. Momentum distributions from a Monte Carlo wavefunction calculation after 100 kicks for $\kappa = 12.5$, $\alpha = 0.0045$, $\bar{k} = 2.1$ and spontaneous emission rate per kick of $\eta = 0.76\%$ (solid), 2.3% (dot–dashed) and 13% (dashed). The lineshapes undergo a transition from exponentially localized to nearly Gaussian. To highlight this the crosses indicate a least-squares quadratic fit of the log of the distribution for $\eta = 13\%$.

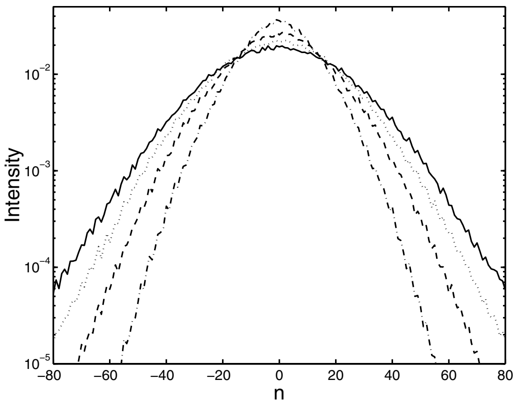

Figure 2. Momentum distributions from a Monte Carlo wavefunction calculation for $\kappa = 11.9$, $\alpha = 0.0141$, $\bar{k} = 2.1$ and spontaneous emission rate per kick of $\eta = 13\%$ after 17 (dot–dashed), 34 (dashed), 51 (dotted) and 68 (solid) kicks.

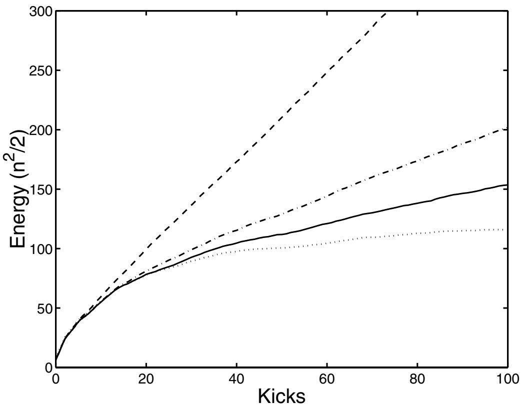

Figure 3. Kinetic energy as a function of the number of kicks from a Monte Carlo wavefunction calculation for $\kappa = 12.5$, $\alpha = 0.0045$, $\bar{k} = 2.1$ and spontaneous emission rate per kick of $\eta = 0\%$ (dotted), 0.76% (solid), 2.3% (dot–dashed) and 13% (dashed). Energy diffusion takes place even when the momentum distributions are little changed.

phase space boundaries at $n \simeq 87$ modify the wings of the momentum distribution for the lowest rate of spontaneous emission $\eta = 0.076\%$. At the lower spontaneous emission rates the distributions quickly become nearly straight lines on this log–linear plot, although both are to some extent rounded near zero momentum, partly as a result of the width of the initial thermal momentum distribution. This is consistent with the essentially exponentially localized momentum distributions observed by the Auckland group [9, 11] at low rates of spontaneous emission. We see that for the 13% spontaneous emission rate, the momentum distribution is significantly different, appearing to be parabolic on the log–linear plot, indicating a nearly Gaussian distribution. Indeed a least-squares fit to a quadratic gives a very much lower error than a fit to a line and this fit is also plotted in figure 1. The shape of this distribution appears to be very similar to the distribution reported for this level of spontaneous emission by the Austin group [10].

The evolution of momentum distributions with the number of kicks is also of interest and was reported by the Austin group. A direct comparison with their results may be made by comparing the evolution of the momentum distributions shown in figure 2, which are calculated for the published parameters of the Austin experiment, with those in figure 1(c) of the erratum of [10].

Figure 3 plots the increase in energy for the various values of spontaneous emission. For the lower rates of spontaneous emission these values are in quantitative agreement with our earlier theoretical and experimental results [8]. The growth of energy occurs even when the shape of the momentum distribution is relatively unaffected by the decoherence. The diffusion rate that is expected due solely to the coupling to the environment (as described by equation (3)) is of the order of $\eta\hbar^2 k_l^2/m$ per kick, or in the units of the figure $\eta/4$ per kick. The observed and the calculated diffusion rates are much greater than this, which implies that it is not simply the energy absorbed from the environment through light scattering that is responsible for the diffusion. The decohering effect of the coupling to the environment is the most significant cause of the increase in the atomic kinetic energy compared with the unitary evolution.

It can be seen from figure 3 that the growth of energy at long times is linear with the number of kicks. As discussed in [7, 8] there is a simple analytic estimate of this long-time diffusion rate $\mathcal{D}_\infty$:

$$\mathcal{D}_\infty = \frac{\eta N^* D_0}{1 + \eta N^*} \tag{6}$$

where D_0 is the short-time diffusion coefficient, which is closely related to the classical diffusion, and N^* is the crossover (or break) time. This formula indicates that for low levels of spontaneous emission the long-time diffusion increases linearly with η and that it saturates for stronger noise. In figure 4 $\mathcal{D}_\infty$, calculated from our simulations, is plotted for various values of η and compared with the value obtained from equation (6). As in our earlier work [8] our new simulations give long-time diffusion rates which agree very well with this formula for appropriate values of D_0, N^*.

The behaviour of the Q-DKR in the presence of noise in the kicking strength is qualitatively very similar. Figure 5

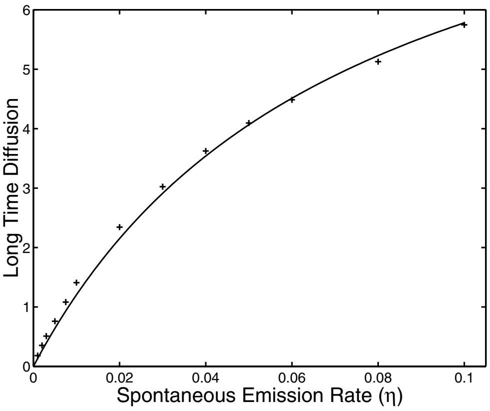

Figure 4. The long-time diffusion $\mathcal{D}_\infty$ calculated from Monte Carlo wavefunction simulations for $\kappa = 11.9$, $\alpha = 0.0045$, $\bar{k} = 2.08$ and various rates of spontaneous emission (crosses). The long-time diffusion predicted by equation (6) is plotted for comparison with $D_0 = 10$, $N^* = 13.7$ (solid curve).

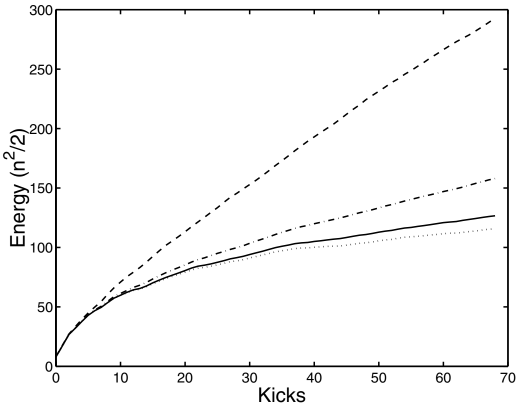

Figure 5. Kinetic energy as a function of the number of kicks from numerical simulation with noise in the kicking strength for $\kappa = 12.8$, $\alpha = 0.0141$, $\bar{k} = 2.1$ and noise of 0% (dotted), 5% (solid), 10% (dot–dashed) and 25% (dashed). These plots represent averages over 20 realizations of the noise process.

plots the kinetic energy as a function of the number of kicks for a range of noise strengths using the parameters of the Austin experiment. Even weak levels of noise lead to diffusion. With increasing levels of noise there is also a transition from exponentially localized to nearly Gaussian momentum lineshapes: figure 6 plots momentum distributions after 68 kicks for the same parameters. This is very similar to the behaviour of the lineshapes under spontaneous emission as in figure 1. The evolution of the momentum distribution for a noise level of 25% is shown in figure 7. These momentum lineshapes appear to be very similar to the ones found experimentally by the Austin group. Again our theoretical calculations are sufficient to reproduce the qualitative behaviour of the Austin experiments; the energy against time graphs do not, however, appear to be in quantitative agreement. This could be due to a number of reasons, the simplifications inherent in our current simulations or an imperfect match to the experimental conditions.

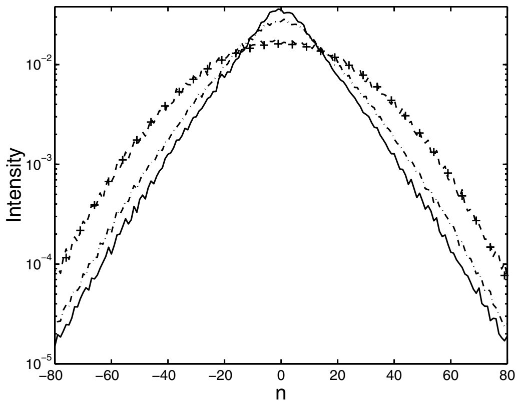

Figure 6. Momentum distributions after 68 kicks from numerical simulation with noise in the kicking strength for $\kappa = 12.8$, $\alpha = 0.0141$, $\bar{k} = 2.1$ and noise of 2.5% (solid), 10% (dot–dashed) and 25% (dashed). Again, the lineshapes undergo a transition from exponentially localized to nearly Gaussian. The crosses indicate a least-squares quadratic fit of the log of the distribution for the 25% noise level.

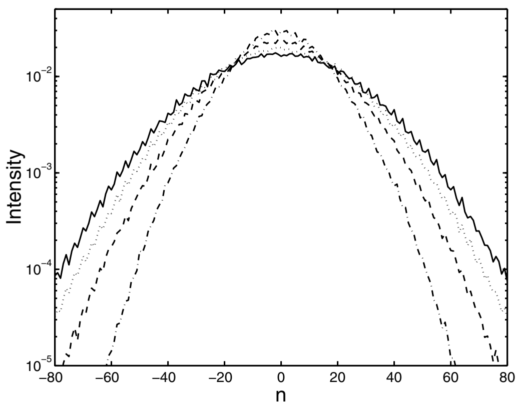

Figure 7. Momentum distributions from numerical simulation with noise in the kicking strength for $\kappa = 12.8$, $\alpha = 0.0141$, $\bar{k} = 2.1$ and 25% noise after 17 (dot–dashed), 34 (dashed), 51 (dotted), and 68 (solid) kicks.

5. Conclusion

In summary, the atom optics manifestation of the Q-DKR offers a unique and pristine environment for studying quantum chaos and decoherence. Much information about the system can be gained through measurements of the kinetic energy growth of the ensemble. However, the shape and character of the momentum distributions is also informative and their observation will continue to be utilized in future quantum chaos experiments. Moreover, there is a need for a more detailed understanding of the specific dynamics of these atom optics experiments and particularly of the nature of the transition from essentially quantum mechanical to essentially classical behaviour. Here we have discussed one approach to the simulation of the experiment based on Monte Carlo wavefunction simulations and a simple initial application of it to a question of relevance to recent experiments. Although we have considered the atomic motional dynamics in a simplified situation these simulations may be straightforwardly generalized to include the full dynamics of the internal state of the atom also.

These calculations have been applied to the specific question of the observed behaviour of the atomic kinetic energy and momentum distributions in the presence of spontaneous emission and of noise in the parameter regimes of the experiments performed by our group [8, 11] and also by the Austin group [10]. We have found that for low levels of spontaneous emission and early times there is a linear rate of increase of kinetic energy without any essential modification of the momentum distributions from those observed for the unitary dynamics. Similar theoretical results were found in the case of a modulated phase potential [13]. For higher levels of noise or longer times the momentum distribution undergoes a transition from exponentially localized to essentially Gaussian. Taking into account the different parameter regimes of each experiment this appears to be in agreement with the experimental results of both groups. The Austin experiment observes qualitatively similar behaviour for both spontaneous emission and a noisy potential and this is again confirmed by our current simulations. Future applications of these techniques could include an investigation of the effect of the parameters κ, $\bar{k}$ and of the noise statistics on the behaviour observed here and in the experiments, particularly for values of these where the classical model predicts anomalous diffusion [21].

The experiments of both groups have demonstrated that decoherence, whether from spontaneous emission or noise, can restore qualitatively classical behaviour to the Q-DKR. The main features of the classical model, energy diffusion and Gaussian momentum distributions are both observed quantum mechanically as long as the noise strength is sufficiently strong. However, until the noise level is very high [12] there is no detailed correspondence in that, for example, the diffusion rate obtained is not that predicted classically. Moreover the levels of decoherence required in the current experiments to lead to apparently classical behaviour are rather large. A fair comparison between the quantum mechanical and classical theories must compare the master equation to a classical DKR with noise [5] for which there is, in any case, extra energy diffusion. At the current levels of noise the classical dynamics of such a noisy DKR are significantly different from the noiseless case. This is perhaps unsurprising in that the experiments operate in a regime in which $\bar{k}$ is rather large, the regime termed 'moderate $\hbar$' in [6]. In this regime, the effect of noise is perturbative, the coherence time of the system in the presence of noise being much longer than the quantum break time [7]. For a sufficiently large rotor, and thus small $\bar{k}$, there should however be a sharper transition to classical behaviour in which the classical diffusion rate is recovered even for noise strengths which have a negligible effect on the classical DKR [6]. This regime deserves further investigation both experimentally and theoretically if it is to be established that decoherence is sufficient to lead to complete correspondence between the quantum mechanical and the classical descriptions of the DKR. Another open question is suggested by the qualitative similarity of thebehaviour of the Q-DKR in the presence of spontaneous emission and of noise. It should be the case that the roughly classical behaviour results simply from the loss of quantum mechanical coherence due to a weak coupling of the rotor, or other chaotic system, to the external world. The details of this coupling should not greatly affect the behaviour of the system. At least for the noise models so far used experimentally, this would appear to be the case. However, a more general understanding of the properties of the coupling that are required to restore classical behaviour is important for future investigations in this field.

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