Delta Kick Cooling: A New Method for Cooling Atoms

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We present a new technique for cooling atoms below the photon recoil temperature. Free expansion and a subsequent application of a pulsed potential narrows the momentum distribution provided the atoms were initially well localized. Time scales for this cooling mechanism are shorter than those for other techniques. We give the one dimensional results for quantum and classical distributions of atoms initially held in an optical lattice or a dipole trap. The pulsed lattice potential is the same as that used in the recent atom optics realization of the quantum delta kicked rotor. [S0031-9007(97)02699-9]

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The application of various optical techniques to cool atoms is an active and important discipline in atomic physics. The study of supercooled atomic gases or the wave nature of matter via atomic optics relies on efficient cooling schemes. The magneto-optic trap [1] and Sisyphus cooling [2] can initially cool the atoms to sub-Doppler temperatures, while Raman cooling [3] and velocity-selective coherent population trapping (VSCPT) [4,5] serve as all optical techniques for achieving subrecoil temperatures; the recoil temperature in terms of the atom's mass m and photon momentum $\hbar k$ is defined by $k_B T_{\rm rec}/2 = (\hbar k)^2/2m$. Evaporative cooling can dramatically increase the phase space density of the atoms. The culmination of this effort was the observation of Bose-Einstein condensation [6,7]. Optical lattices [8] and optical dipole traps [9] are used both to hold atoms at fixed locations and to cool them further when coupled with adiabatic [8,10], evaporative [11], or Raman [12] cooling schemes.

The cooling technique described in this Letter originated from the study of chaotic systems. The quantum delta kicked rotor (DKR) has provided insight into basic issues of quantum systems which exhibit chaotic behavior in the classical regime [13–15]. It is normally understood that during the initial evolution of the rotor its energy grows diffusively, be it in the classical or quantum domain. It may therefore be surprising that with a judicious choice of free expansion time and kicking strength, the energy of the rotor may be considerably reduced after the first kick. The condition for this to occur is an initial localization of the rotor state in configuration space.

The work of Moore *et al.* provided a bridge between the DKR and atom optics. In order to observe dynamical localization they exposed ultracold Na atoms to a pulsed optical lattice [14]. With this atomic optics realization of the DKR one can envisage a new cooling technique for atoms. The above mentioned initial condition for energy extraction from the rotor is an initial localization of the atom to within a fraction of one optical wavelength. We show that atoms under these conditions can be further cooled by what we call the delta kick cooling (DKC) technique. The time scale for DKC is shorter than that required for adiabatic [8,10], evaporative [11], Raman [12], or VSCPT [5] cooling schemes. Free expansion for some μs, followed by a single delta kick (actually of the order of a few hundred ns), will produce temperatures of a few times the recoil limit. Temperatures far below the recoil limit can be obtained by applying DKC to atoms initially held in an optical dipole trap. Ultimately the cooling limit is set by spontaneous emission. Our analysis is restricted to one dimension. DKC conserves phase space density.

Let us introduce the basic idea behind DKC in general terms, starting with classical considerations and then presenting a quantum argument. Let a single atom be described by the Hamiltonian $H_0 = p^2/2m + U(x)$, which determines the atom's dynamics for $t < 0$. At $t = 0$ we extinguish the potential $U(x)$ and subsequently pulse it on again after a certain free expansion time T. Multiplying $U(x)$ with a Gaussian pulse in time $\exp[-(t - T)^2/2\tau_p^2]$, and assuming a short enough pulse width τ_p, the Hamiltonian for $t > 0$ becomes

$$H_k = \frac{p^2}{2m} + V(x)\delta(t - T), \tag{1}$$

where $V(x)$ is related to $U(x)$ by $V(x) = \sqrt{2\pi}\,\tau_p U(x)$. Note that the kick strength can be varied by choosing an appropriate pulse width. A classical ensemble of atoms localized around the bottom of $U(x)$ will, upon its extinction, expand and atoms with differing momentum will separate in space. After a long enough time T the momentum of the atoms will essentially be a linear function of x, $p = mx/T$, a fact exploited in time of flight measurements. Assuming that after free expansion the atomic cloud is still confined to some region R, where $U(x)$ can be approximated by a harmonic potential $U(x) \approx m\omega^2 x^2/2$ within R, a kick will change the momentum of each atom by $\Delta p \propto dU/dx \propto x \propto p$. By setting the pulse width to an appropriate value dependent on the free expansion time, all the atoms will be nearly at rest after the kick. This condition is fulfilled if $\kappa_{\rm cl} \approx 1$ where

$$\kappa_{\rm cl} = \sqrt{2\pi}\,\tau_p \omega^2 T. \tag{2}$$

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There are two main reasons why the atoms' velocity after the kick will not be perfectly zero. First, the ensemble has a finite initial spread in space with the effect that $p = mx/T$ will be only approximately fulfilled, the approximation improving with better initial localization and longer free expansion time T. Second, the kicking potential will only be harmonic within a finite region in space R, which in turn limits the free expansion time T. There is obviously a trade-off for T, and as a rule of thumb one can state the following: The time T should be chosen such that the atomic cloud "covers" the region R before applying the kick.

The quantum mechanical argument basically yields the same results. The time evolution operator $\hat{U}_F$ and the kick operator $\hat{U}_K$ are derived from the Hamiltonian Eq. (1), yielding $\hat{U}_F = \exp[-i\hat{p}^2 T/2m\hbar]$ and $\hat{U}_K = \exp[-i\hat{V}/\hbar]$. Starting with an atom in the ground state (a minimum uncertainty wave packet) of $U(x)$ which is then shut off, $\hat{U}_F$ broadens $|\psi(x)|^2$; the phase of $\psi(x)$ will be a quadratic function of x after free expansion. The following kick does not affect the magnitude of $\psi(x)$, but influences its phase. Matching the corresponding phase change $V(x)/\hbar$ to the phase the atom has picked up during free evolution again produces a near minimum uncertainty wave packet, but as the width of $|\psi(x)|^2$ has broadened by a factor Q, $|\phi(p)|^2$ will be narrowed by the same factor Q and the energy will be reduced roughly by Q^2. As in the classical picture, the energy reduction will be more pronounced with longer T, with a limit set by the finiteness of the region R. One can derive a quantum expression for the matching condition as in the classical case, which yields $\kappa_{\rm cl}(1 + 1/\omega^2 T^2) \approx 1$, showing their equivalence for large enough T.

It is interesting to note that the reduction in the momentum distribution is formally similar to pulse compression in optics with a fiber-grating pair [16]. There the phase of the optical pulse changes due to self-phase modulation (analogous to the atoms' free expansion shift), while matching this phase to that induced by anomalous group velocity dispersion of a grating pair (analogous to the kick) produces the compression.

As a first example, and as an illustration of these general statements we will now apply DKC to atoms initially held in an optical lattice [8,10], which for one dimension is intimately connected to the kicked rotor appearing in the quantum chaos literature [14]. Two counterpropagating laser beams (wave number k_l and frequency ω_l) produce a standing wave. We assume a two level atom (transition frequency ω_0), and a detuning $\Delta = \omega_0 - \omega_l$ sufficiently large compared to the natural linewidth. The standing wave produces a periodic potential of the form $U(x) = (-U_0/2)\cos(2k_l x)$, where $U_0 = \hbar\Omega^2/4\Delta = \hbar\Omega_{\rm eff}/4$ and Ω is the Rabi frequency for a single beam. With these definitions, the frequency in the harmonic approximation becomes $\omega = \sqrt{\omega_{\rm rec}\Omega_{\rm eff}}$, where $\omega_{\rm rec} = \hbar k_l^2/2m$. Given $U(x)$ one can write down $V(x)$ and Eq. (1) becomes the celebrated Hamiltonian for the kicked rotor (with the difference that the delta kick occurs just once). It is amusing to note that the classical stochasticity parameter κ that traditionally appears in the chaos literature and which governs whether the system's behavior will be chaotic or regular is just our $\kappa_{\rm cl}$ [13–15]. For optimal cooling $\kappa_{\rm cl}$ is close to one, which implies that the corresponding kicked rotor system is at the border between integrability and chaos [15].

Consider a minimum uncertainty Gaussian state within the lattice potential $U(x)$, $\langle x^2\rangle = \sigma_x^2$ and $\langle p^2\rangle = \hbar^2/4\sigma_x^2$. Let the atoms expand freely for a time T, followed by the delta kick. Immediately after the kick one can show that the momentum spread is

$$\langle p^2\rangle = \frac{\hbar^2}{4\sigma_x^2} + \frac{\kappa_{\rm cl}^2 m^2}{8k_l^2 T^2}\left[1 - e^{-8(\sigma_x^2 k_l^2 + \hbar^2 T^2 k_l^2/4\sigma_x^2 m^2)}\right] - \frac{\hbar^2\kappa_{\rm cl}}{2\sigma_x^2} e^{-2(\sigma_x^2 k_l^2 + \hbar^2 T^2 k_l^2/4\sigma_x^2 m^2)}. \quad (3)$$

An appropriate choice of T and $\kappa_{\rm cl}$ produces a reduction in the momentum width. Figure 1 displays an example for Cs atoms ($\lambda = 852$ nm) initially in the ground state of a potential with a depth of $U_0 = 10^3\, U_{\rm rec}$ ($U_{\rm rec} = \hbar^2 k_l^2/2m$). After free expansion for 3.55 μs, followed by a kick with $\kappa_{\rm cl} = 1.14$ ($\tau_p = 190$ ns), the final momentum spread is reduced from $\langle p^2\rangle = 15.8(\hbar k_l)^2$ to $\langle p^2\rangle = 2.48(\hbar k_l)^2$. Note that the optimal $\kappa_{\rm cl}$, which is readily obtained by minimizing the expression given in Eq. (3), is slightly larger than one, meaning that an optimal kick will actually overcompensate the phase of the wave function. This can be seen in Fig. 1(b), where the phase of the final wave function is shown to exhibit a small positive curvature at the center; also in Fig. 1(a) one can recognize small bumps in the final momentum distribution, very similar to those of compressed optical pulses. Another subsequent kick will only marginally reduce the kinetic energy.

A classical analysis predicts the same reduction in the momentum distribution width. Classically, the free expansion followed by the delta kick is just the standard

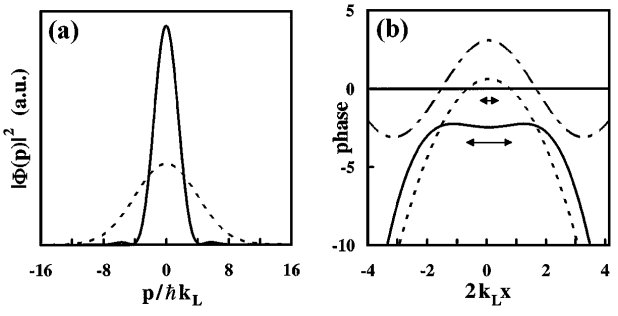

FIG. 1. (a) Initial momentum distribution (dashed curve) for Cs atoms with $\sigma_x = 2.0 \times 10^{-2}\lambda$, $\langle p^2\rangle = 15.8(\hbar k_l)^2$, and final distribution [solid curve, $\langle p^2\rangle = 2.48(\hbar k_l)^2$]. (b) Phase of the wave function after free expansion (dashed curve) and after kick (solid curve), as well as the periodic potential (dot-dashed curve). Arrows indicate spatial FWHM before and after free expansion.

map [15]. Starting with a phase space probability density characterized by the same widths as those of $|\psi(x)|^2$ and $|\phi(p)|^2$, followed by free expansion for T and the delta kick, the resulting momentum width is again given by Eq. (3). Figure 2 displays the evolution of the classical phase space density, from its initial value, to after free expansion, and then after the kick. The free expansion stretches the distribution in the x direction, but leaves the energy of the ensemble unchanged. The kick then "rotates" the elongated distribution by an angle in phase space, which minimizes the final energy. The above mentioned overcompensation is displayed by the fact that the long axis of the final distribution is not strictly parallel with the x axis. This reduces the energy in the tails which arise from a deviation of the cosine potential from the harmonic potential for larger x values. A longer free evolution time would make the tails more pronounced, thereby increasing the energy.

In the quantum calculation up to now we have considered only the lattice ground state. This is equivalent to neglecting an initial thermal excitation of the atoms held in the lattice. Assumed in what follows are Cs atoms thermally distributed in the modes of the trapping potential. For the temperatures used the populated states are reasonably approximated by those of a harmonic oscillator. We start with an initial density matrix $\hat{\rho} = e^{-\hat{H}/k_B T}/Z$, where H is the harmonic oscillator approximation of the lattice Hamiltonian and Z the partition function. The final density matrix is calculated by well known formulas using the appropriate free evolution and the kick operator, $\hat{U}_F$ and $\hat{U}_K$, respectively, where in $\hat{U}_K$ the true lattice Hamiltonian is used (not its harmonic approximation). Figure 3 displays the results of a numerical calculation for an optical lattice with a well depth of $U_0 = 10^3 U_{\rm rec}$. Instead of optimizing the expansion time and kick strength for each initial temperature, we show the final temperatures for three different but fixed sets of parameters, which correspond to the optimized values for initial temperatures of $60T_{\rm rec}$, $30T_{\rm rec}$, and below $15T_{\rm rec}$ ($T_{\rm rec} = 200$ nK for Cs). Not surprisingly, the optimized free expansion time is longer for smaller initial temperatures: 2.2, 2.7, and 3.6 μs. The corresponding pulse widths are 290, 240, and 190 ns; the degrees of overcompensation $\kappa_{\rm cl}$ are 1.06, 1.11, and 1.14. In Fig. 3 the initial lattice temperature determines the Bolzmann distribution of the population of states. A value of $T = 0$ implies all atoms are in the ground state of a lattice well. Extinguishing the lattice leaves atoms with a net kinetic energy and associated translational temperature which is then reduced by the action of the kick. We also note that adiabatic cooling produces lower temperatures than DKC. The time scale for DKC is much shorter.

As discussed above, the condition for cooling atoms by DKC relies on their initial localization in configuration space to a fraction of R. Traditional optical dipole traps [9] may also serve as a means of initially holding the atoms. We will show that the lowest temperatures achievable via DKC utilize dipole traps. Consider Na atoms moving in the transverse direction with respect to a red detuned laser beam, which is focused to a waist size $w = 12$ μm, providing a potential of the form $U(x) = U_0[1 - \exp(-2x^2/w^2)]$ with a well depth of $U_0/k_B = 2$ mK $= 830T_{\rm rec}$ ($T_{\rm rec} = 2.4$ μK for Na). The atoms are assumed to be thermally distributed in the modes of

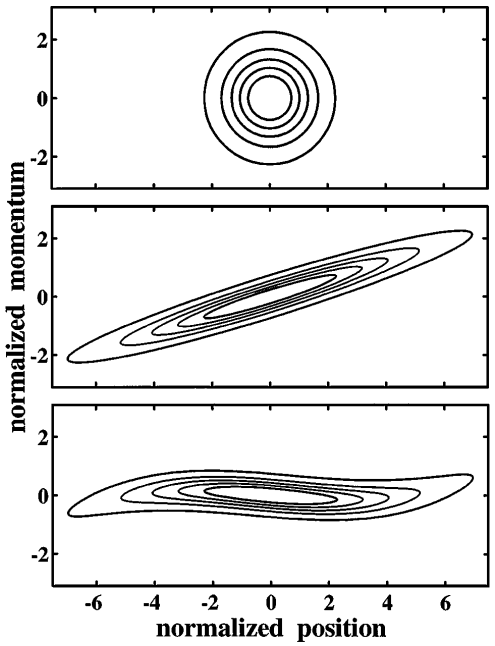

FIG. 2. Evolution of the classical phase space density: initial (top), after free expansion (middle), and after the kick (bottom). Same parameters as in Fig. 1.

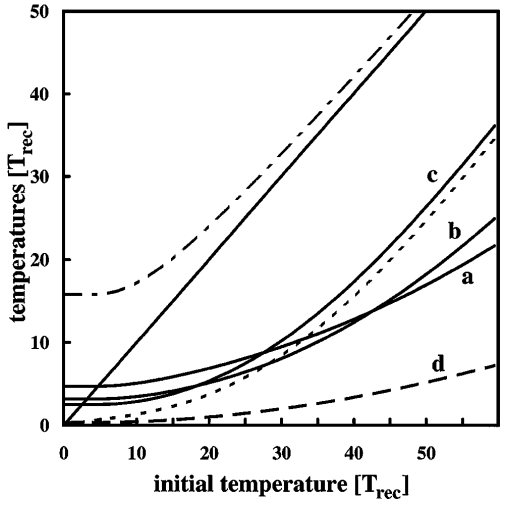

FIG. 3. Final versus initial temperature for Cs atoms in an optical lattice; well depth of $U_0 = 10^3 U_{\rm rec}$. No temperature change (solid curve) and translational temperature after instantaneous elimination of the lattice (dot-dashed curve); the two curves will coincide for large temperatures. DKC results optimized for initial lattice temperatures of $60T_{\rm rec}$ (curve *a*), $30T_{\rm rec}$ (curve *b*), and less than $15T_{\rm rec}$ (curve *c*, classical analysis dotted curve). Adiabatic cooling [8] (curve *d*).

the harmonic approximation of the trapping potential, the calculation of the final temperature being essentially the same as the one described in the lattice section. Figure 4 displays the results, again for three different but fixed T and τ_p obtained by minimizing the final temperature for initial temperatures of $10T_{\rm rec}$, $1T_{\rm rec}$, and below $0.1T_{\rm rec}$. In the $10T_{\rm rec}$ case, a free expansion time of 27.4 μs and a pulse duration of $\tau_p = 850$ ns reduces the temperature down to $T_{\rm rec}$, while for an initial temperature of $T_{\rm rec}$ [17], a free expansion time of 57 μs and pulse time of $\tau_p = 385$ ns reduces the temperature to $2.4 \times 10^{-2} T_{\rm rec}$. For initial temperatures less than $0.1T_{\rm rec}$, a free expansion time of 94 μs and pulse time of $\tau_p = 224$ ns, DKC reduces the temperature to $2 \times 10^{-3} T_{\rm rec}$.

One could envisage other similar DKC scenarios. Reaching ultralow temperatures in the conventional dipole trap described above depends on the difficult task of effectively Raman cooling atoms within a strong potential [17]. This could possibly be overcome by using a more novel, blue detuned dipole trap which confines the atoms in a free-space-like environment, where efficient Raman cooling has recently been demonstrated [12]. The trap could be eliminated and after a fixed time a pulse from another beam, of appropriate size and intensity, would flash on. Still another possibility would consist of using a traditional dipole trap as described, but increasing the region R by using a second kicking beam with a larger beam waist. Consider for example a red detuned optical dipole trap for Na atoms with a single beam focused to a 12 μm waist, a potential well depth of $U_0/k_B = 2$ mK, with the atoms at $T_{\rm rec}$. The trap is then shut off and the atoms expand for a longer period of time, say 140 μs. A second beam is then pulsed on. The free evolution phase will match the curvature of the kicking potential if one chooses, for example, a beam waist of 40 μm and a potential depth of $U_0/k_B = 0.9$ mK (1 W peak power, $\Delta\lambda = 4.8$ nm) and a pulse width of $\tau_p = 3.70$ μs. This results in cooling the atoms down to a temperature of $5 \times 10^{-3} T_{\rm rec}$. For this example the pulse still reasonably approximates a delta kick as the relative displacement of the atoms during the kick is of the order of 10^{-2}. The average probability for atomic excitation is 10^{-3}, indicating that scattering of DKC light will ultimately limit the expansion time: Longer T require larger beam waists and thus, as the power is limited, a smaller detuning thereby increasing the photon absorption rate. It has yet to be determined whether longer pulses might be suited for DKC as well, which would somewhat relax the above stated restrictions. We note that specifically designed kicks, in the spirit of the work of Mielnik [18], may provide other interesting examples of wave function manipulation.

The kicked rotor is a system that has always been associated with energy gain when viewed classically or quantum mechanically. As shown in this Letter, a state initially well localized in position space may have its momentum distribution narrowed. Atoms localized in single wells of an optical lattice can be cooled to near $T_{\rm rec}$ by eliminating the lattice for a few μs, then pulsing it on again for some hundreds of ns. Optical dipole traps used similarly, but with free expansion times in the order of 100 μs, may ultimately cool atoms to below $10^{-2} T_{\rm rec}$.

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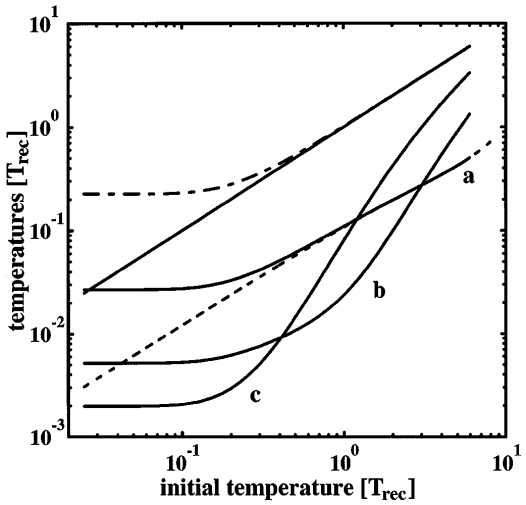

FIG. 4. Final versus initial temperature for Na atoms in an optical dipole trap; well depth of $U_0/k_B = 2$ mK. No temperature change (solid curve). Temperature after instantaneous elimination of dipole trap (dot-dashed curve). DKC results optimized for initial temperatures of $10T_{\rm rec}$ (curve *a*, classical analysis dashed curve), $T_{\rm rec}$ (curve *b*), initial temperatures less than $0.1T_{\rm rec}$ (curve *c*).

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