Mixing internal and external atomic dynamics in the kicked rotor

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A modified atomic version of the quantum δ-kicked rotor is considered. In contrast to previous studies, we do not eliminate the excited-state amplitude. This gives rise to a mixing of internal and translational atomic dynamics. A semiclassical argument suggests that this could conceivably result in momentum delocalization. However, our analysis shows that the added complexity fails to destroy dynamical localization.
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I. INTRODUCTION

In spite of the well-known arguments supporting the nonexistence of genuine quantum chaos [1], a few models with positive Lyapunov exponents have been reported. An example is the many-particle quantum system considered by Parmenter and Yu [2]. One peculiarity of their model is the functional dependence of a certain effective Hamiltonian on the wave function. This situation presumably does not occur in more elementary single-particle problems. We note, however, that quantum chaos has also been reported in comparatively simple models, for example, in Weigert's cat map [3], but there the kinetic energy of the particle grows exponentially with time and many would consider such an unbounded explosion as unphysical. Is quantum chaos consequently possible only in systems that possess an ample complexity? What degree of complexity is necessary in order for quantum chaos to occur?

The answer to these questions is not known; it is not even clear whether they are the right questions to ask. If they are, then it is certainly still worthwhile to look for quantum chaos in systems with a modest degree of complexity. The simplest classical chaotic system is indisputably the δ-kicked rotor (DKR) [4]. The fact that its quantum counterpart shows diffusion in momentum space only until the so-called quantum break time but dynamical localization afterward means that only a limited number of Floquet states are involved in the dynamics, which in turn entails quasiperiodicity and ergo lack of chaos. Dynamical localization is therefore often termed ''suppression of chaos,'' implying that there would be chaos if there was not localization. We do not agree with this notion, if only in the sense that delocalization is certainly necessary (but by no means sufficient) for chaotic behavior in quantum DKR models. It has been shown that delocalization can be achieved by adding noise [5] to the system, but this introduces randomness by itself and is as such not in the spirit of classical chaos theory. More impressive is the restoration of diffusive energy growth by a periodic, incommensurate modulation of the kick strength [4,6]. However, this still does not introduce sensitive dependence (as can be seen, for example, from the time reversibility of the dynamics in a computer experiment).

In this paper we will add some more complexity to the quantum DKR without sacrificing the strict periodicity of the driving force and without introducing noise. The atomic realization of the quantum DKR [7] offers such a possibility if one takes the dynamics of the internal atomic state into account. So far, in all related work, the excited-state population of the atom was assumed to be negligible by virtue of a large laser-atom detuning. One is then left with the wave function for the atomic ground state alone and the internal and external dynamics are decoupled. It is well known, however, that in general the electronic and translational degrees of freedom of an atom moving in an optical potential cannot be separated from each other [8]. This mixing gives rise to the extremely rich and beautiful behavior observed with atoms suspended in near-resonance light fields. In order to gain some insight into the effect of such mixing on the dynamical localization, we have chosen to study a modification of the atomic realization of the DKR, one that is simple enough such that the Floquet operator can be constructed analytically.

We will consider a two-level atom in a classical standing wave, where the latter is turned on and off periodically (a sequence of short square pulses) and spontaneous emission is neglected. The analysis is therefore similar to atomic beam deflection in optical standing waves [9]. Then, pictured semiclassically, the effect of a ''kick'' on the atom depends not only on its position, as in the DKR, but also on its internal state. If the time evolution of the latter does not have the same periodicity as the pulse sequence (or if it is not periodic at all, perhaps even irregular), then one would expect this situation to be comparable to a modulated DKR. Therefore, delocalization and a higher degree of dynamical irregularity would not come as a surprise.

The structure of this paper is as follows. In Sec. II we introduce our model of a two-level atom within a periodically modulated one-dimensional optical lattice. The results for a typical example are presented in Sec. III. A comparison with the DKR is given in Sec. IV. Finally, a summary and some ideas for further studies are contained in Sec. V.

II. ATOMIC DYNAMICS

Let us consider a two-level atom with ground state $|g\rangle$, excited state $|e\rangle$, and transition frequency ω_0. In order to simplify the description of the atom in the light field we change to the interaction picture $|\tilde{g}\rangle = \exp(i\omega t/2)|g\rangle$ and $|\tilde{e}\rangle = \exp(-i\omega t/2)|e\rangle$, with ω being the frequency of the external field. Further, we assume the detuning to be small Δ

$=\omega-\omega_0 \ll \omega_0$. Neglecting spontaneous emission, the dipole and rotating-wave approximations yield the familiar Schrödinger equation [8]

$$i\hbar\,\frac{\partial\tilde{\psi}}{\partial t}=\left(\frac{p^2}{2m}-\frac{\hbar\Delta}{2}\,\sigma_3-\frac{\hbar\,\omega_1(x,\,t)}{2}\,(\sigma^+ +\sigma^-)\right)\tilde{\psi},$$

$$\tilde{\psi}=\begin{pmatrix}\tilde{\psi}_e(x,\,t)\\ \tilde{\psi}_g(x,\,t)\end{pmatrix}, \tag{1}$$

where $p^2/2m$ is the kinetic energy associated with the atomic center-of-mass momentum p and the term with $\omega_1(x,t)=2\Omega\,\cos(kx)f(t)$ describes the coupling between the atom and a prescribed classical standing wave, Ω is the Rabi frequency corresponding to a single traveling wave, $k=\omega/c$ is the wave number, and the function $f(t)$ accounts for turning the external field on and off. The time dependence of $f(t)$ is taken to be a periodic sequence of square pulses of duration τ and pulse spacing T such that $f(t)=1$ for $mT\leqslant t<mT+\tau$ $(m=0,1,2,\ldots)$ and $f(t)=0$ otherwise. We further restrict ourselves to the case of short interaction times $\tau\ll 1/kv$, where v is an upper bound for the velocity that the atom acquires in this course of time. This condition implies that during the interaction time the atom does not move appreciably compared to the laser wavelength and the operator of the kinetic energy in Eq. (1) can be discarded (Raman-Nath approximation) [9]. Under this condition, it is possible to explicitly calculate the operators $\hat{U}_I$ and $\hat{U}_F$, which specify the evolution of the atomic state during the interaction time and the free atomic evolution between the light pulses, respectively. The Floquet operator $\hat{U}$ is then simply given by $\hat{U}=\hat{U}_F\cdot\hat{U}_I$. In order to construct $\hat{U}_I$, we proceed as follows. At $t=0$, the state $\tilde{\psi}(0)$ is projected onto the eigenstates of the Hamiltonian in Eq. (1) neglecting the kinetic-energy term (''dressed states'') [8,10]. As time evolves, the amplitudes in this basis simply pick up a phase that is proportional to the interaction time and the corresponding eigenenergy. Note that both the eigenvectors and the eigenenergies are position dependent, although the variable x appears just as a parameter. Then we project the state $\tilde{\psi}(\tau)$ back onto the basis in the interaction picture. The procedure is straightforward and yields for the matrix elements of the operator $\hat{U}_I$

$$\langle\tilde{e}\,|\hat{U}_I|\tilde{e}\,\rangle=\cos\left(\frac{\Omega_{\rm eff}\tau}{2}\right)+i\,\frac{\Delta}{\Omega_{\rm eff}}\sin\left(\frac{\Omega_{\rm eff}\tau}{2}\right), \tag{2a}$$

$$\langle\tilde{e}\,|\hat{U}_I|\tilde{g}\,\rangle=i\,\frac{\omega_1}{\Omega_{\rm eff}}\sin\left(\frac{\Omega_{\rm eff}\tau}{2}\right), \tag{2b}$$

$\langle\tilde{g}\,|\hat{U}_I|\tilde{g}\,\rangle=\overline{\langle\tilde{e}\,|\hat{U}_I|\tilde{e}\,\rangle}$, and $\langle\tilde{g}\,|\hat{U}_I|e\rangle=\langle\tilde{e}\,|\hat{U}_I|\tilde{g}\,\rangle$. The effective Rabi frequency is given by $\Omega_{\rm eff}(x)=\sqrt{\omega_1^2(x)+\Delta^2}$. Note that the spatial period of the diagonal elements of $\hat{U}_I$ is equal to $\lambda/2$, whereas for the off-diagonal elements it is λ. This implies that a state $\delta(p-p_0)|\tilde{g}\,\rangle$ only couples to states $\delta(p-p_0-2j\hbar k)|\tilde{g}\,\rangle$ and $\delta(p-p_0-(2j+1)\hbar k)|\tilde{e}\,\rangle$, where j is an integer (the same is true if $\tilde{e}\leftrightarrow\tilde{g}$). The free evolution operator $\hat{U}_F$ is diagonal and can be easily written down in the momentum representation

$$\langle\tilde{e}\,|\hat{U}_F|\tilde{e}\,\rangle=\exp[-in^2\omega_r(T-\tau)+i\Delta(T-\tau)/2], \tag{3a}$$

$$\langle\tilde{g}\,|\hat{U}_F|\tilde{g}\,\rangle=\exp[-in^2\omega_r(T-\tau)-i\Delta(T-\tau)/2], \tag{3b}$$

where we have set $n=p/\hbar k$ and the recoil frequency ω_r, is given by $\omega_r=\hbar k^2/2m$.

Equations (2) and (3) fully determine the time evolution of an initial atomic state, although it is not possible to write down the general solution for an arbitrary number of periods T analytically. After a rescaling t/T, we are left with four model parameters τ/T, $\omega_r T$, ΩT, and ΔT. In order to facilitate the comparison with the DKR, we change to the set of parameters

$$\alpha=\tau/T, \tag{4a}$$

$$\overline{k}=8\,\omega_r T, \tag{4b}$$

$$\kappa_g=4\,\alpha\,\omega_r\Omega^2T^2/\sqrt{\Omega^2+\Delta^2}, \tag{4c}$$

$$\beta=\Omega/\Delta. \tag{4d}$$

Here α and β are parameters that do not appear in the DKR model, where the pulse is assumed to be a δ kick $(\alpha\to 0)$ and the detuning in the atomic realization of the DKR is taken to be much larger than the Rabi frequency $\beta\ll 1$. It is easy to check that for $\alpha,\beta\ll 1$ our model reduces to the system considered in Ref. [7] if one replaces n by $n/2$ (in the DKR literature n corresponds to an atomic momentum $p=n2\hbar k$). We will see below that with the definition of the generalized ''kicking strength'' κ_g of Eq. (4c), the energy diffusion at an early stage of the time evolution is proportional to κ_g^2 as is the case for the DKR. Note finally that in the new notation the condition for the Raman-Nath approximation reads $n\ll 8/\alpha\overline{k}$, which can be fulfilled for any n by choosing α small enough.

III. RESULTS FROM A TYPICAL EXAMPLE

We have calculated the time evolution of a given initial atomic state numerically by using Eqs. (2) and (3) and applying a fast Fourier transform algorithm. In all our calculations, the momentum axis was descretized in steps of the photon momentum $\hbar k$. The behavior of the system was analyzed for a broad range of parameter values. As an illustration, consider he case where we have $\kappa_g=11$, $\overline{k}=1.5$, $\alpha=0.005$, and $\beta=0.13$. Note that κ_g dictates the degree of classical chaos in the limiting case $\alpha,\beta\ll 1$. Momentum diffusion is obtained only for large enough kicking strengths, such that the corresponding phase-space structure is globally chaotic. As is well known, in the classical version of the DKR the last Kolmogorov-Arnold-Moser boundary is destroyed for $\kappa_g\gtrsim 1$. Further, the parameter $\overline{k}$ has the usual meaning of an effective Planck constant and the pulse width α is set to be small enough to guarantee the validity of the Raman-Nath approximation. Finally, the parameter β is a measure of the degree of internal atomic excitation and is chosen in this representative example to yield an intermediate upper-state population.

Initially, we assume the atom to be in the internal ground state. Its translational state is taken to be a coherent Gaussian

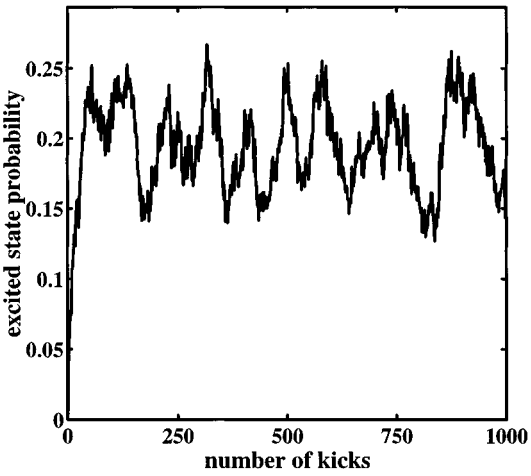

FIG. 1. Probability of the two-level atom occupying the excited state as a function of the number of kick and free evolution cycles. The parameters used were $\kappa_g = 11$, $\overline{k} = 1.5$, $\alpha = 0.005$, and $\beta = 0.13$.

wave packet (on the above-mentioned grid in momentum space), which is centered around $n = 0$ with a full width at half maximum of $\Delta n = 7$. Figure 1 displays the upper-state probability as a function of the number of kicks. As can be seen, the probability fluctuates around 20% and creates a situation that could conceivably have a chance of destroying localization, at least according to the semiclassical reasoning in the Introduction. However, it turns out that dynamical localization persists despite the mixing of the internal and external dynamics. Figure 2 illustrates the situation after the evolution of the wave function through 1000 kick and free evolution cycles. As can be clearly seen, both the ground and excited momentum distributions have become exponentially localized. Apparently, our semiclassical arguments were too simplistic.

The transition from momentum diffusion to localization is displayed in Fig. 3. We have plotted $\langle n^2/2 \rangle$ for the ground and the excited state as well as the total kinetic energy of the atom as a function of time. For all cases the momentum

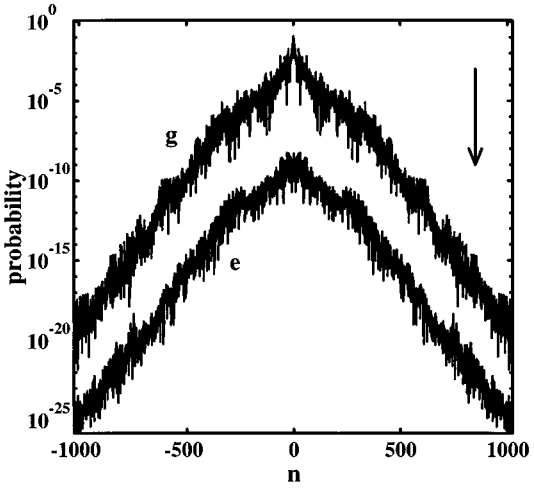

FIG. 2. Ground- and excited-state momentum probability distributions; both are exponentially localized. For convenience the excited-state distribution was displaced down by the amount indicated by the arrow on the upper right.

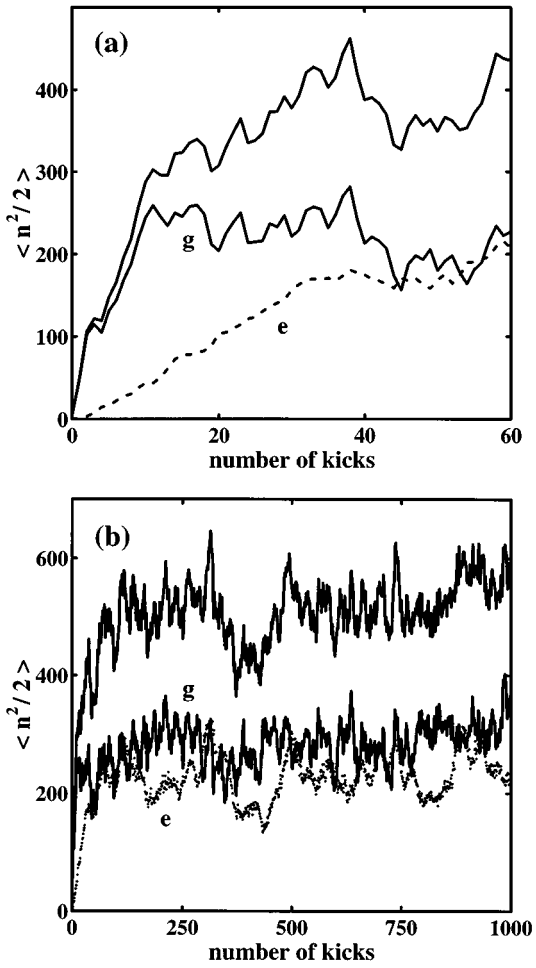

FIG. 3. Kinetic energy for the ground state and the excited state vs the number of kicks: (a) 60 kicks and (b) 1000 kicks. The top trace is the total kinetic energy.

width rapidly increases at an early stage of the time evolution, but localization eventually sets in. To conclude, we mention that the contribution of the excited state to the total kinetic energy is almost as large as the one from the ground-state wave function, although the upper-state probability is only 20%. Therefore, the upper-state momentum distribution is, on the average, less localized than its ground-state counterpart.

IV. COMPARISON WITH THE δ-KICKED ROTOR

While our model is strictly not a direct analog of the DKR, a comparison can still be made. As was already mentioned, the system reduces to the DKR when $\alpha, \beta \ll 1$, so one can compare our model to the DKR by examining the kinetic energies at an equivalent value of kick strength κ_g. Because our model tracks the behavior of a two-level atom, with a wave function that is a superposition of ground and excited states, it has no direct classical counterpart. The DKR limit provides us with a means to connect our model to the classically chaotic regime.

In Fig. 4 we have plotted the total kinetic energy versus the number of kicks for the parameter set $\kappa_g = 7$, $\overline{k} = 1$, $\alpha = 0.005$, and three different values of β. The initial conditions were the same as in Sec. III. Trace 1 has $\beta = 10^{-5}$, so

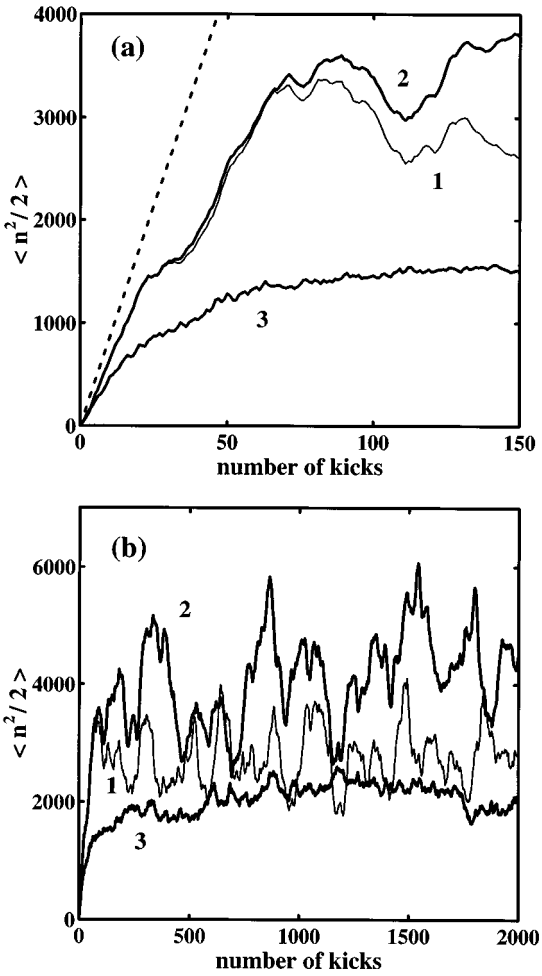

FIG. 4. Total kinetic energy vs the number of kicks for $\kappa_g=7$, $\overline{k}=1.0$, and $\alpha=0.005$: (a) 150 kicks and (b) 1000 kicks. Traces 1–3 have $\beta=10^{-5}$, 0.04 and 10, respectively. The dashed line is the classical DKR energy diffusion.

small that the system is in fact very close to the DKR; its excited-state probability is less than 10^{-7}. This DKR-like case can now be compared with situations where the interplay between electronic and translational degrees of freedom plays a more significant role. For $\beta=0.04$ (trace 2), the energy growth closely follows the one corresponding to negligible excitation during the first ~ 70 kick and free evolution cycles. This time is needed for the upper-state population to build up (not shown) before it saturates at approximately 15%, while fluctuations similar to the ones displayed in Fig. 1 persist. For large values of β (trace 3, $\beta=10$), the excited-state probability reaches a value slightly above 50% right after the first kick. Consequently, the energy in Fig. 4 deviates from the DKR behavior much earlier than observed in the previous case of medium excitation. However, in spite of these deviations, one can still distinguish between a momentum diffusion and a localization regime even for this case of strong mixing. One further aspect is noteworthy for this latter case. If the degree of excitation is large, we have noticed that the upper-state probability always fluctuates quite erratically around 50% and that the corresponding autocorrelation functions decay rapidly. This is not true for intermediate or low excitation.

While the details of the traces displayed in Fig. 4 depend critically on the model parameters chosen, we can state the following with respect to a comparison of our two-level system with mixed dynamics with the conventional DKR. Neither the diffusive energy growth rate nor the saturation level depends markedly on the degree of internal atomic excitation. This implies that the formula for the diffusion rate, which is well established within the DKR model, is applicable to the model considered here too. We finally would like to point out that a saturation of the two-level atom's momentum spread was not observed when the quantum resonance condition was fulfilled [11]. For $\overline{k}$ an integer multiple of 4π, the kinetic energy grows quadratically with time just as with the quantum DKR. However, this result is of moderate relevance for an atomic system, as the quadratic increase is expected to be absent if the restriction to integer values of $\hbar k$ on the momentum axis is lifted, which for an atom moving in one dimension would be the proper thing to do [7].

It is interesting to note that decoherence between the ground and excited state alone (i.e., without affecting the coherence of the translational degree of freedom) destroys localization. As a first naive modification of our model, we randomly changed the phase between the two internal states after every kick. Then the internal degrees of freedom may act as an ''environment'' coupled to the translational motion. Our preliminary calculations indeed indicate that if the random phase changes are sufficiently large then the momentum diffusion mimics the classical evolution. The effects of decoherence on this two-level system are a topic of continued further research.

V. CONCLUSION

In summary, we have analyzed the behavior of a two-level atom in a time-dependent classical standing wave, where the internal atomic dynamics have been taken into account. For negligible population of the upper state, the model reduces to the previously investigated atomic realization of the DKR [7]. By adding this extra degree of freedom, semiclassical arguments and an analogy to the modulated DKR [4,6] suggest that the dynamical localization observed in the quantum DKR could possibly be destroyed and one could thus hope for more irregularity in the corresponding dynamics. However, our numerical studies have shown that the localization mechanisms are apparently more robust than anticipated. Irrespective of the atomic initial conditions, the interaction strength, and the degree of internal excitation, one always observes the characteristic exponential envelope for the momentum probability distribution. This is somewhat startling when one considers the quite erratic time dependence of the electronic excitation and the fact that the force experienced by an atom in the light field depends on its internal state. Surprisingly, the additional complexity of an atom with internal dynamics fails to destroy dynamical localization.

The work presented in this paper could be extended down a number of different avenues. It would be interesting to see

whether the abolition of the Raman-Nath approximation and the retention of nonadiabatic terms essentially changes any of our results. Also interesting would be studies on the effect of optical polarization gradients and the inclusion of more realistic atomic models with complex hyperfine structure and corresponding Zeeman sublevels. However, we suspect that regardless of these possible modifications, a sensitive dependence on initial conditions, a characteristic trait of classical chaos, will still be absent.

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