ARTICLES

Optimal detection strategies for measuring the stochastic gravitational radiation background with laser interferometric antennas

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(Received 24 April 1996)

Issues pertaining to the optimal strategy for detecting the stochastic gravitational wave background (SGWB) with laser interferometric antennas are discussed. Analyzed are the dependence of detection sensitivity on the relative orientation of interferometers, the interferometer design, and the inherent noise of the detectors. Previously Michelson, Flanagan, and Christensen thoroughly studied such topics. This paper addresses a few remaining issues for the optimal detection of the SGWB with laser interferometers. The optimal orientation of a pair of interferometers depends on both the noise characteristics of the detectors and their physical location on the surface of the Earth. Given a pair of detectors the maximum sensitivity for detecting the SGWB also depends on the transfer function of the interferometers; the relatively narrow band dual recycling interferometers are the best choice. Correlated noise in two antennas located at a single site complicates the detection strategy, but an optimistic attitude is called for given the considerable relative size of the correlated signal. The Laser Interferometric Gravitational Wave Observatory offers exciting prospects for placing limits on the strength of the SGWB. [S0556-2821(97)01102-8]

PACS number(s): 04.80.Nn, 04.30.Db, 98.80.Es

I. INTRODUCTION

In a few years a number of collaborations around the world will be operating laser interferometric gravitation radiation antennas. In the United States the Laser Interferometric Gravitational Wave Observatory (LIGO) is under construction, with 4-km arm length interferometers in Hanford, Washington, and Livingston, Louisiana [1]. Similar detectors may be built in Europe [2] and Australia [3]. One of the most intriguing sources of gravitational radiation could be from the events in the early Universe. This radiation will pervade our space time as a noise on the background metric. Detection of the stochastic gravitational wave background (SGWB) would provide physicists with extremely useful cosmological information. References [4–6] discuss possible sources and their strengths.

The magnitude of the SGWB is expected to be extremely small. It is essential that the optimum strategy for detection is understood. Even limits on the strength of the SGWB will have important cosmological implications. A worldwide network of laser interferometric antennas can provide useful and important limits on the SGWB [6]. As a consequence, a complete understanding of all the detection issues is vital. Michelson [7] provided the first comprehensive study on the extraction of the SGWB signal from the correlated output of two quadrupole detectors. Christensen [6,8] discusses many topics related to an efficient strategy for the laser interferometric detection of the SGWB. Flanagan [5] recently published a thorough analysis on the topic. While the results presented by Flanagan [5] are elegant and impressive, some of the details of some of the conclusions are worthy of discussion; this paper attempts to address these conclusions.

The discussion of the detection of the SGWB differs from that of other gravitational radiation sources in that it is more sensible to talk in the language of the spectrum of the energy density of the radiation, and not in terms of the amplitude of the waves. The spectrum of the SGWB may range from frequencies as cosmologically low as $1/T_{\rm Hubble}$ to as high as a thermal (1 K black body) 10^{11} Hz [8]. It is useful to express the SGWB in terms of the ratio of the gravity wave energy density per logarithmic frequency interval to the closure density of the Universe, ρ_c: namely,

$$\Omega_{\rm GW}(f)=\frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ \ln f}, \tag{1.1}$$

where $\rho_{\rm GW}$ is the energy density of the SGWB. It is not unreasonable to expect advanced interferometers, such as those planned for LIGO, to limit the SGWB around 100 Hz to $2\times10^{-10}\ \rho_c$ [6]; this would be a limit on the SGWB averaged over the bandwidth of the interferometer. LIGO [1], and similar systems under consideration around the world [2,3], offer exciting prospects for making significant cosmological discoveries [4,6].

In the simplest scenario the output data stream from two detectors will be optimally filtered, then multiplied together, and averaged over a long time period. Assuming that the noise in each interferometer is uncorrelated with the noise in the other, and assuming that both the signal and noise are stochastic and stationary [9], then the signal-to-noise ratio (SNR) for a measurement of the SGWB is [5–7]

$$\left(\frac{S}{N}\right)^2=\left(\frac{4G\rho_c}{5\pi c^2}\right)^2 2\tau\int_0^\infty df\,\frac{\Omega_{\rm GW}(f)^2\gamma(f)^2}{f^6 S_n(f)^2}, \tag{1.2}$$

where τ is the integration time, $S_n(f)$ is the spectral density of the noise for each detector, and $\gamma(f)$ is the overlap reduction function (normalized to unity for optimal alignment). The $\gamma(f)$ term accounts for the reduction in sensitivity due to detector separation and alignment [5–7]. The problem of maximizing detection sensitivity is equivalent to maximizing the integral in Eq. (1.2). In order to do this one must make an assumption about the character of $\Omega_{\rm GW}(f)$. A reasonable assumption would be that $\Omega_{\rm GW}(f)$ is roughly constant in the frequency band of the detectors; the analysis in this paper will utilize this assumption, as did the analysis of Ref. [5]. The remaining parameters to adjust in Eq. (1.2) are $\gamma(f)$ and $S_n(f)$. Given the locations for a detector pair one could, in principle, adjust the orientations of the interferometers and also modify the noise spectral density via a modification in the interferometer transfer function in the shot noise dominated regime. One will always attempt to decrease the low-frequency noise of the detector, but thermal and seismic noise will be a nuisance below 50 Hz. The optimal detection strategy depends on the antennas' orientation and transfer function; the tradeoffs between these effects will be discussed. A relatively narrow band measurement utilizing dual recycling interferometers [10] always provides the maximum SNR; this is contrary to the assumption made in Ref. [5].

The previous work of Flanagan [5] and Christensen [6] addresses the strategy for choosing the optimal orientation of two interferometers. The optimal orientation is always one of the following: in configuration I one arm of each interferometer lies along the great circle that joins the detectors, while for configuration II each detector arm is at an angle of 45° to the great circle. It was initially stated [6] that the optimal alignment for detecting the SGWB was always configuration I. The exact closed-form solution for $\gamma(f)$ derived by Flanagan showed that configuration II could often provide a better solution, and it was stated that this is true when the interferometer pair subtends an angle at the center of the Earth of $\leqslant$70° [5]. Emphasized in this present paper is the fact that the optimum configuration is highly dependent on the noise characteristics of the interferometer, but that ultimately the difference in sensitivity provided by either configuration I or II is insignificant. The alignment issues are important when one considers the potential construction of a worldwide network of interferometers.

Also highlighted in this paper is the importance of attempting a correlation measurement with two interferometers at the same site. The LIGO system will ultimately have multiple interferometers within the same vacuum system [1]. Correlated noise will certainly frustrate the extraction of the SGWB signal; it was correctly noted that correlated noise that appears out of phase in each antenna will dramatically effect the correlation experiment [5]. In Ref. [5] the same site correlation measurement was discounted. An important point to be highlighted here is that this same site correlation experiment should be encouraged. While the believability of such an experiment will certainly be problematic to achieve, the relatively large correlated signal [manifesting itself with $\gamma(f)=1$ for all frequencies] should force one to drop pessimistic attitudes. The potential correlated noise sources have been highlighted elsewhere [6,8]. The confirmation of the detection of any gravity wave source (periodic, burst, or stochastic) will be difficult to achieve. The study of all interferometer noise sources, correlated or not, will prove to be a necessary exercise during the operation of the antennas. Studies and active monitoring of noise will provide extremely useful information needed for the confirmation of the detection of all gravity wave sources.

The organization of this paper is as follows. Section II highlights how assumptions on the broadband noise signature of the interferometers influences the optimal alignment. Section III provides an analysis of the improvement in the SNR by use of dual recycling techniques. Section IV contains the discussion of issues pertaining to measuring the SGWB via a correlation from two interferometers at a single site, and within a single vacuum system. Section V is the conclusion.

II. OPTIMUM ALIGNMENT AND DETECTOR NOISE

The elegant closed-form solution for the overlap reduction function, $\gamma(f)$, derived by Flanagan [5] has considerably simplified the SGWB detection analysis. In the discussion below the angle β is defined as that angle subtended by two detectors at the center of the Earth. The optimum alignment of interferometers on the surface of the Earth for detecting the SGWB is always one of the following: one arm of each interferometer lies along the great circle connecting the two antennas (configuration I), or each interferometer arm makes an angle of 45° to this line joining the detectors (configuration II). For configuration I the solution is

$$\gamma(f)=\frac{1}{4}\left[(1+\cos^2\beta)\rho_1(\alpha)+\cos\beta\,\cos^2\frac{\beta}{2}\rho_2(\alpha)+\cos^4\frac{\beta}{2}\rho_3(\alpha)\right], \qquad (2.1)$$

while, for configuration II,

$$\gamma(f)=\frac{1}{4}\left[2\cos\beta\rho_1(\alpha)+\cos^2\frac{\beta}{2}\rho_2(\alpha)\right]. \qquad (2.2)$$

The $\rho_i(\alpha)$ terms are functions of the spherical Bessel functions [5,11]:

$$\rho_1(\alpha)=5j_0(\alpha)-10j_1(\alpha)/\alpha+5j_2(\alpha)/\alpha^2, \qquad (2.3)$$

$$\rho_2(\alpha)=-10j_0(\alpha)+40j_1(\alpha)/\alpha-50j_2(\alpha)/\alpha^2, \qquad (2.4)$$

and

$$\rho_3(\alpha)=5j_0(\alpha)/2-25j_1(\alpha)/\alpha+175j_2(\alpha)/(2\alpha^2), \qquad (2.5)$$

where the argument is a function of the detector separation distance, $\alpha=4\pi f\,\sin(\beta/2)R_\oplus/c$.

In order to make a direct comparison with Ref. [5] the assumed noise power spectral density for the interferometers will be approximated as

$$S_n(f)=\max[S_m(f/f_m)^{-4},S_m(f/f_m)^2]. \qquad (2.6)$$

A value of $S_m=10^{-48}\ \mathrm{Hz}^{-1}$ and $f_m=70$ Hz (with a cutoff at 10 Hz) approximates the long term goal for the advanced, broadband, LIGO interferometers [1]. These are the values used in the analysis by Flanagan [5]. The low-frequency noise of interferometers is, and will certainly continue to be, the nemesis of gravity-wave physicists. It would be extremely useful for gravity-wave detection (burst, periodic, or stochastic signals) if the interferometers could operate in Hz or sub-Hz frequency bands, but thermal and seismic noise will certainly contaminate any low-frequency measurement. If the interferometers can actually operate such that they are shot noise limited down to 70 Hz with 60 W of laser power it will be an experimental achievement of the most remarkable and laudable kind. As noted by Flanagan [5], configuration II is more sensitive to the $\rho_2(\alpha)$ term in $\gamma(f)$; the assumption of the constant value of $\Omega_{GW}(f)$ and the low-frequency (70 Hz) sensitivity of the interferometers makes configuration II the better solution for $\beta \leqslant 80°$. This is displayed in Fig. 1. Figure 1(a) shows the normalized SNR [as defined by Eq. (1.2)] as a function of β for configurations I and II. Figure 1(b) displays the relative difference between the two solutions: namely,

$$\frac{(S/N)_{\rm I}-(S/N)_{\rm II}}{(S/N)_{\rm I}+(S/N)_{\rm II}}, \tag{2.7}$$

as a function of β.

In the near future it is likely that operational gravity-wave interferometers will only be shot noise limited down to about 200 Hz. For comparison, Figs. 2 and 3 display the normalized SNR when the assumed noise spectral density was changed by modifying f_m in Eq. (2.6) to 150 and 200 Hz, respectively. These noise corners are more realistic; even accomplishing shot noise limitation at 200 Hz with 5 W of laser power will be a significant accomplishment. Note now that when the noise elbow is at 150 Hz, configuration II is the optimum alignment for $15° \leqslant \beta \leqslant 50°$, and configuration I is optimal otherwise. When the noise elbow is at 200 Hz, configuration II is the optimum alignment for $15° \leqslant \beta \leqslant 40°$, and configuration I is optimal otherwise. When the low-frequency response of the interferometer's sensitivity begins at the relatively larger frequencies (say above 150 Hz) it is the domination of the $\rho_1(\alpha)$ term in configuration I's $\gamma(f)$, and to a lesser extent the absence of $\rho_3(\alpha)$ in configuration II's $\gamma(f)$, that typically makes configuration I the optimum solution. For all practical purposes the relative difference between the two configurations is only a few percent; realistically the two alignments offer the same sensitivity to the SGWB. This conclusion can also be drawn from Flanagan's results [5].

It should be noted that configuration II's sensitivity is quite remarkable. Consider the extreme situation where $\beta = 90°$. As the frequency of the gravity waves approaches zero we have $\gamma \to 0$. For this alignment $\gamma(f)$ depends only on $\rho_2(\alpha)$. For low frequencies these two interferometers are effectively orthogonal to each other, as displayed by Fig. 6 of Forward's classic paper [12]. Yet, as the frequency of the waves approaches about 100 Hz, the effect of the $\rho_2(\alpha)$ term produces a nontrivial cross correlation.

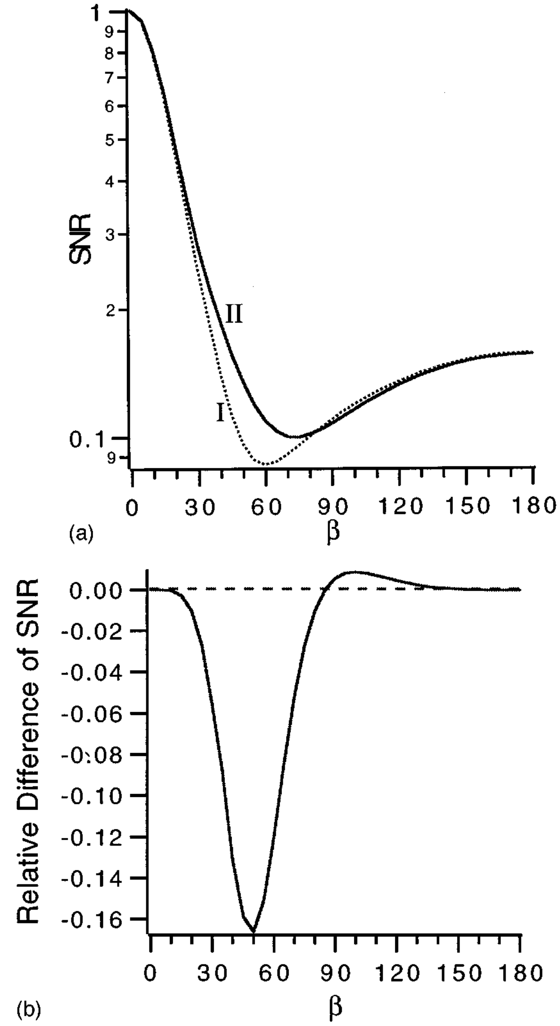

FIG. 1. (a) The normalized signal-to-noise ratio for a correlation experiment between two detectors as a function of the angle β subtended between them at the center of the Earth. The noise spectral density for the broadband interferometers is assumed to be shot noise limited down to 70 Hz. For configuration I an arm of each of the interferometers lies along the great circle connecting the detectors, while for configuration II each interferometer arm is at 45° to this arc. (b) The relative difference between the signal-to-noise ratios for configurations I and II, as a function of β. Specifically, a plot of $[(S/N)_{\rm I}-(S/N)_{\rm II}]/[(S/N)_{\rm I}+(S/N)_{\rm II}]$ for the values displayed in (a). Note that configuration II offers the best alignment for $\beta \leqslant 80°$.

III. BROADBAND VERSUS NARROW-BAND MEASUREMENTS

When one assumes that the gravity-wave energy density per logarithmic frequency interval is constant then the f^6 term in the denominator of Eq. (1.2) weights the integral in favor of low-frequency signals. From this Flanagan [5] concluded that the optimal strategy for detecting the SGWB would be to use broadband detectors, as opposed to narrow-band devices, such as the dual recycling interferometers [10]. Presented in this section are results displaying the fact that dual recycling will in fact increase the SNR.

For the analysis it was assumed that Fabry-Pérot interferometers were used, with an arm length of 4 km. The reflectivities of the cavity mirrors on the central test masses [1] are $R_1 = 0.9221$, while the reflectivities of the mirrors on the test masses at the far end of the cavities are $R_2 = 0.9995$; this yields a storage time for the cavities of $\tau_s = 6.67 \times 10^{-4}$ s, or

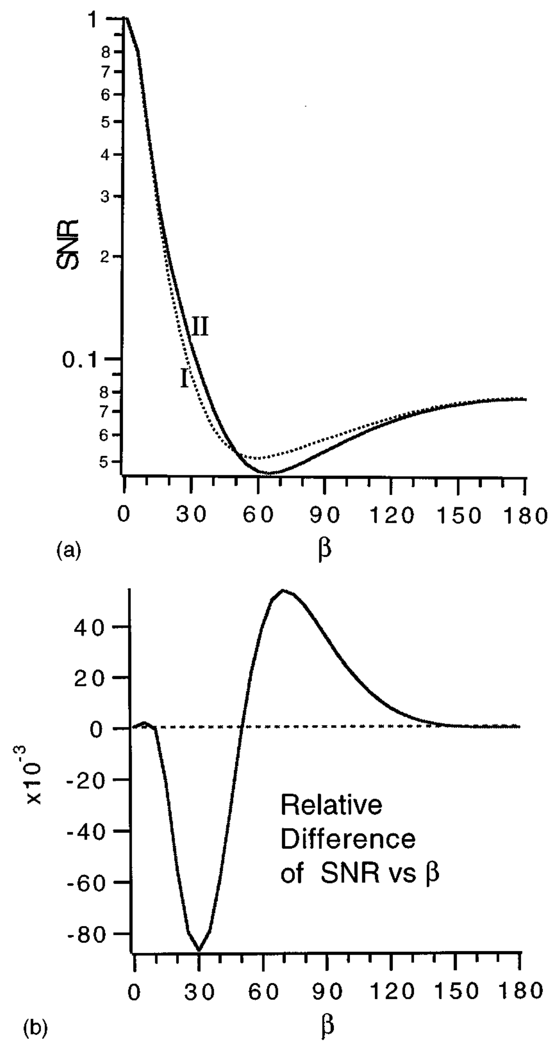

FIG. 2. (a) The normalized signal-to-noise ratio for a correlation experiment between two detectors as a function of the angle β subtended between them at the center of the Earth. The noise spectral density for the broadband interferometers is assumed to be shot noise limited down to 150 Hz. (b) The relative difference between the signal-to-noise ratios for configurations I and II, as a function of β. Configuration II offers the best alignment for $15° \leqslant \beta \leqslant 50°$.

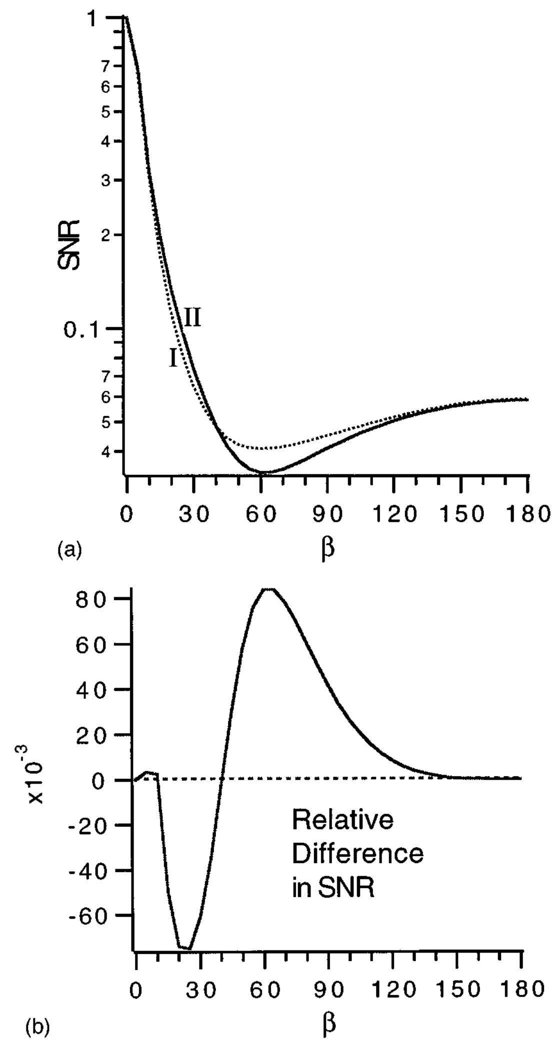

FIG. 3. (a) The normalized signal-to-noise ratio for a correlation experiment between two detectors as a function of the angle β subtended between them at the center of the Earth. The noise spectral density for the broadband interferometers is assumed to be shot noise limited down to 200 Hz. (b) The relative difference between the signal-to-noise ratios for configurations I and II, as a function of β. Configuration II offers the best alignment for $15° \leqslant \beta \leqslant 40°$.

50 effective bounces. A recycling mirror is used to feedback the laser light that would normally exit the interferometer towards the laser [1]; for these cavities one derives an optimum value of $R_0 = 0.9705$. The dual recycling mirror, R_3, will feedback the signal light that exits the interferometer towards the photodetector [1,10]. All optical elements were assumed to have a loss of 10^{-4}. The noise spectral density of the interferometers were assumed to be the sum (in quadrature) of shot noise (60 W laser power at $\lambda = 514.5$ nm) and a seismic-thermal noise term that varied as $S_{s-t}(f) \propto (f/f_m)^{-4}$, where, for broadband recycling (i.e., $R_3 = 0$) the two noise terms were equal in magnitude at f_m. The transfer functions for the standard recycling and dual recycling Fabry-Pérot interferometers can be found elsewhere [8,10].

Figure 4 shows the normalized SNR for configuration II for a broadband recycled interferometer, and a dual recycling system. Assumed for the noise was $f_m = 70$ Hz; the seismic-thermal noise spectral density was the same for both systems. The reflectivity of the dual recycling mirror (R_3) was chosen so as to maximize the SNR, with the resonant frequency for this narrow-band detector chosen to lie at the location of the local maximum of $\gamma(f)^2$ nearest to 100 Hz. For example, Fig. 5 shows the optimum transfer function for a dual recycling system for configuration I ($\beta = 45°$, $R_3 = 0.79$) along with $\gamma(f)^2$; the vertical was modified so that the two curves touch at the resonant frequency of 104.3 Hz. For the dual recycling results displayed in Fig. 4 the optimum reflectivity fell within the range $0.70 \leqslant R_3 \leqslant 0.80$. For $\beta \approx 0$ dual recycling produced an increase in the SNR of 1.9 over the broadband recycling result. With dual recycling systems having a noise corner of $f_m = 70$ Hz and the resonant frequency chosen to lie at the location of the local maximum of $\gamma(f)^2$ nearest to 100 Hz, configuration II offered the best alignment for detecting the SGWB for the angles of $20° < \beta < 65°$. It should still be noted that for all angles the two alignment results never differed by more than a few percent.

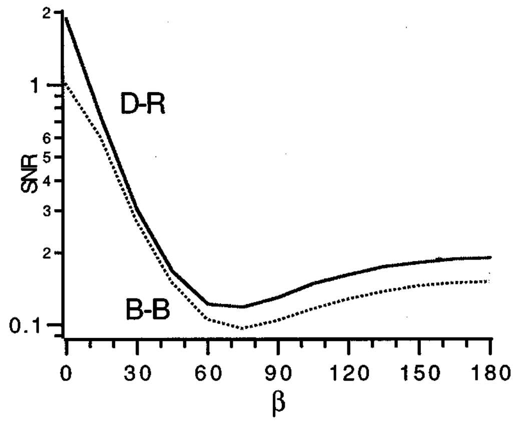

FIG. 4. The normalized SNR (as a function of β) for configuration II for a pair of broadband recycled interferometers (labeled B-B on graph), and a pair of dual recycling systems (labeled D-R). The shot noise spectral density for broadband recycling equaled the seismic-thermal noise at $f_m=70$ Hz. The reflectivity of the dual recycling mirror (R_3) was chosen so as to maximize the SNR, with the resonant frequency for this narrow-band detector chosen to lie at the location of the local maximum of $\gamma(f)^2$ nearest to 100 Hz.

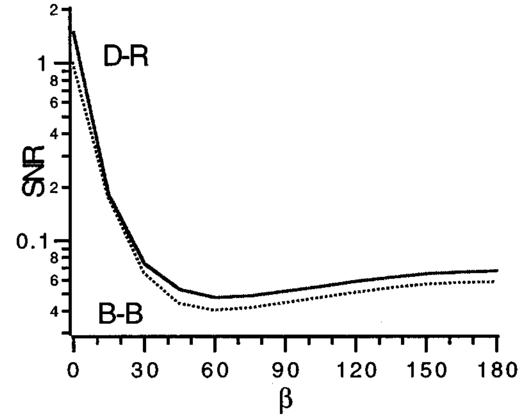

FIG. 6. The normalized SNR (as a function of β) for configuration I for a pair of broadband recycled interferometers (labeled B-B on graph), and a pair of dual recycling systems (labeled D-R). The shot noise spectral density for broadband recycling equaled the seismic-thermal noise at $f_m=200$ Hz. The reflectivity of the dual recycling mirror (R_3) was chosen so as to maximize the SNR, with the resonant frequency for this narrow-band detector chosen to lie at the location of the local maximum of $\gamma(f)^2$ nearest to 200 Hz.

For a noisier system, with $f_m=200$ Hz, Fig. 6 displays the broadband versus dual recycling results. Configuration I is used. For the dual recycling interferometers the reflectivity of the dual recycling mirror (R_3) was chosen so as to maximize the SNR, with the resonant frequency for this narrow-band detector chosen to lie at the location of the local maximum of $\gamma(f)^2$ nearest to 200 Hz. For $\beta\approx0$ dual recycling produced an increase in the SNR of nearly 1.5 over the broadband recycling result. For dual recycling systems having a noise corner of $f_m=200$ Hz and the resonant frequency chosen to lie at the location of the local maximum of $\gamma(f)^2$ nearest to 200 Hz, configuration II offered the best alignment for detecting the SGWB for the angles of $15°<\beta<35°$. It should again be noted that for all angles the two alignment results never differed by more than a few percent.

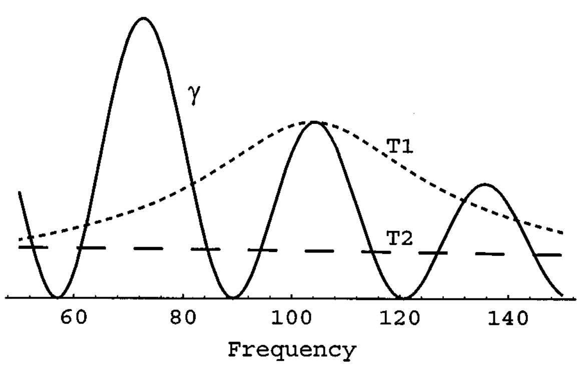

FIG. 5. The optimum transfer function for a dual recycling system for configuration I (β=45°, $R_3=0.79$, labeled $T1$ on graph) along with $\gamma(f)^2$ (labeled γ on graph). A modified vertical scale ensures that the two curves touch at the resonant frequency of 104.3 Hz. Also displayed for comparison is the transfer function for the broadband recycled interferometer (labeled $T2$ on graph), all factors equivalent to those used for $T1$ except now $R_3=0$. Here dual recycling provides a SNR gain of 1.12 over broadband recycling.

For the dual recycling results displayed in Fig. 6 the optimum reflectivity always had a value of $R_3\approx0.67$.

Some characteristics of dual recycling interferometers help to display why this narrow-band technique can be marginally better than the broadband alternative. The dual recycling interferometer's noise power spectral density can be characterized by three features: the bandwidth Δf, the central frequency f_c, and the value of the noise power spectral density at f_c, $S_n(f_c)$. The value of $\Delta f/S_n(f_c)$ is roughly constant as the reflectivity of the dual recycling mirror is changed [10]. Hence in a search for burst waves the signal-to-noise ratio is also independent of bandwidth, as the ratio

$$\left(\frac{S}{N}\right)^2_{\text{burst}}\propto\frac{\Delta f}{S_n(f)} \tag{3.1}$$

remains approximately constant [4]. Examination of Eq. (1.2) shows that for the stochastic background

$$\left(\frac{S}{N}\right)^2_{\text{stochastic}}\propto\frac{\Delta f}{S_n(f)^2}. \tag{3.2}$$

The signal-to-noise ratio increases as the dual recycling interferometer's transfer function is made narrower.

IV. SINGLE-SITE CORRELATION EXPERIMENT

The LIGO system will ultimately contain at least two interferometers within the vacuum system at each site, one detector with 4 km arms and the other with 2 km [1]. For two antennas at the same location and mutually aligned the overlap reduction function, $\gamma(f)$, will equal one for all frequencies. This greatly enhances the SNR, as seen via Eq. (1.2). Flanagan [5] makes the very credible point that correlated noise in two interferometers at a single site, which for some reason appears out of phase in one detector as compared to the other, could potentially cast such uncertainty into the SGWB detection experiment as to make it impossible. While acknowledging that results from such an experiment will be difficult to confirm, it should be stressed that it will not be impossible. The single site correlation experiment should not, at this premature time, be discounted.

The investigation, measurement, and characterization of noise in the interferometers will be an unavoidable task for scientists interested in identifying periodic, burst, and stochastic sources of gravitational radiation. The search for the SGWB depends on a correlated signal, hence a correlated noise source masks the signal. The existence of an out of phase, but correlated, noise in the two antennas could prevent even the possibility of setting a limit on the strength of the SGWB [5]. Potential correlated noise sources were analyzed and discussed extensively in Refs. [6,8].

Consider the data streams from full and half length interferometers at a singles site. The ith component of the data time series takes the form

$$x_{1i}=s_i+n_{1i}, \qquad x_{2i}=\frac{1}{2}s_i+n_{2i}, \tag{4.1}$$

where s is the signal and n the noise. Assume that s and n are independent stationary Gaussian processes with zero mean. The correlation experiment attempts to measure the variance of s, σ_s^2, assumed much smaller than the noise vacancies, σ_{n1}^2 and σ_{n2}^2. The correlation between channels 1 and 2 is [6]

$$r_{12}\cong\frac{(1/2)\sigma_s^2+\rho\sigma_{n1}\sigma_{n2}}{\sigma_{n1}\sigma_{n2}}. \tag{4.2}$$

Flanagan [5] notes that the noise correlation coefficient, ρ, could be nonzero and negative, thereby preventing the possibility of placing a limit on σ_s.

The likely factors to contribute to ρ are seismic noise, fluctuations in the residual-gas column density inside the common vacuum system, and electromagnetic field fluctuations [6]. Unknown sources are likely. Each noise source has its own signature and spectral density. One can easily assume that the contribution that each of these effects makes to ρ will have its magnitude, and probably sign too, dependent on frequency. The frequency dependence of the noise spectral density can help to differentiate between a correlated signal originating from the SGWB and that created by local noise. One can expect the magnitude of the spectral density from most noise sources to vary significantly within the 50 Hz to 1 kHz bandwidth, whereas a SGWB spectral density that changes dramatically in this band requires exotic physics to explain. Albeit cumbersome, the prudent avenue for a single-site correlation experiments is to make a number of narrow-band measurements within the 100 Hz to 1 kHz band. These types of measurements are likely to occur anyway during the search for periodic and burst sources.

A number of narrow-band measurements can place a limit on the strength of the SGWB. While one can imagine correlated noise that appears out of phase between two detectors, the prospect of this occurring across the operating bandwidth of the detectors is unlikely. Use of multiple narrow-band measurements can identify and account for noise; a measurement of a correlated signal within some frequency band that does not appear at other frequencies can certainly be attributed to correlated noise. As an example, we can consider Eq. (2.6) defining noise spectral density of the interferometers with $f_m=70$ Hz. Consider a dual recycling system designed so as to maximize the gravitational wave strain sensitivity at 100 Hz ($R_3=0.975$ and other parameters the same as used above); the full width at half maximum (FWHM) of this interferometer's transfer function is 12.5 Hz [8,10]. The SNR gain of this dual recycling system over a broadband recycling system, as defined via Eq. (1.2), is 1.355 (where the seismic-thermal noise contribution is assumed to be the same for the two cases). This dual recycling system offers very narrow-band resolution, and multiple measurements in the 100 Hz to 1 kHz range can provide information on possible correlated signals plus correlated noise.

The two sites for the LIGO interferometers [1], plus an eventual worldwide network of detectors [2,3], bolster the prospects for single-site correlation experiments. This is especially true for setting believable limits on the strength of the SGWB. While a negative correlation coefficient (within some frequency band) for the noise could hinder the confidence in a measurement at a single site, comparison of results from an identical experiment at another location provides valuable information. The character of correlated noise should vary from location to location; local seismic waves propagate through different types of ground and local electromagnetic fields should differ. Identical same-site correlation measurements made at two separate locations will offer valuable additional information; this is a topic requiring further study.

At this point it is extremely difficult to predict whether intrasite correlations will be useful. Correlated noise sources are hard to predict. A noise correlation coefficient, ρ, as low as 10^{-3} could mask a signal [6]. Individual experiments to determine the sign and phase of ρ at specific frequencies may turn out to be an inappropriate exercise; the integration times required will be long and it may be prudent to just try a single broadband measurement. However, these questions will likely become more understandable once the actual interferometers are operating. At this point in time one should still explore various strategies for extracting the SGWB from an intrasite correlation experiment.

V. CONCLUSION

Addressed in this paper were issues related to developing the optimal strategy for detecting or placing a limit on the strength of the SGWB. Michelson [7], Christensen [6], and Flanagan [5] previously explored this topic in detail; some of the details of some of the conclusions given in Ref. [5] were further analyzed here. Presented in this paper were results showing the optimum alignment of two laser interferometric antennas depends on both their location and noise spectral densities. In the end, configurations I and II differ very little in their results, each is essentially equivalent as the optimum alignment for detecting the SGWB. It is also shown here that narrow-band detectors, such as dual recycling interferometers [10], offer the best hope for detecting the SGWB. The optimum mirror reflectivity for the dual recycling mirror (R_3) will differ from the optimum reflectivity for gravity-wave strain sensitivity. Significant gains in the SNR, up to a factor of 1.9, were displayed in this paper. Finally, encouragement is given to attempts to measure the SGWB with two interferometers at the same site. While this experiment is not easy, it would seem uncharacteristic of gravity-wave physicists to discount its potential. A series of narrow-band, dual recycling, measurements in the 100 Hz to 1 kHz band should help to distinguish the effects of correlated noise from signal. Identical measurements at another location will add confidence, and should be further considered.

The LIGO antennas, plus others to come around the world, offer exciting prospects for making cosmologically significant measurements of gravitational radiation. It still seems realistic to assume that LIGO could place a limit on the strength of the SGWB to be $\Omega_{\mathrm{GW}}(f) < 2 \times 10^{-10}$ at 100 Hz after 10^7 s of integration time [6]. A worldwide network of antennas offers exciting prospects for experimental cosmology.

ACKNOWLEDGMENTS

Grants from the University of Auckland Research Committee partially supported this work.

[1] A. Abramovici, W. E. Althouse, R. W. P. Drever, Y. Gürsel, S. Kawamura, F. J. Raab, D. Shoemaker, L. Sievers, R. E. Spero, K. S. Thorne, R. E. Vogt, R. Weiss, S. E. Whitcomb, and M. E. Zucker, Science **256**, 325 (1992).

[2] C. Bradaschia *et al.*, Nucl. Instrum. Methods Phys. Res. A **289**, 518 (1990).

[3] *The Detection of Gravitational Waves*, edited by D. G. Blair (Cambridge University Press, Cambridge, England, 1991).

[4] K. S. Thorne, in *300 Years of Gravitation*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987), p. 330.

[5] E. E. Flanagan, Phys. Rev. D **48**, 2389 (1993).

[6] N. Christensen, Phys. Rev. D **46**, 5250 (1992).

[7] P. F. Michelson, Mon. Not. R. Astron. Soc. **227**, 933 (1987).

[8] N. L. Christensen, Ph.D. thesis, Massachusetts Institute of Technology, 1990.

[9] J. S. Bendat, *Principles and Application of Random Noise Theory* (Wiley, New York, 1958).

[10] B. J. Meers, Phys. Rev. D **38**, 2317 (1988).

[11] A typographical error [Eq. (B6) of Ref. [5]] gave the wrong value for the second term in the definition of $\rho_1(\alpha)$. The results in Ref. [5] obviously use the correct value, as displayed via its Fig. 10.

[12] R. L. Forward, Phys. Rev. D **17**, 379 (1978).