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The Effects of Classical Barriers in a Quantum Chaos Experiment Using Laser-Cooled Atoms

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Abstract—We experimentally and theoretically study the effect of Kolmogorov–Arnol'd–Moser cantori on momentum diffusion in a quantum system. Cesium atoms which have been cooled in a magneto-optic trap are subjected to a specifically designed periodically pulsed standing wave. A cantori separates two chaotic regions of the classical phase space. Diffusion through the cantori is classically predicted, but for a quantum system the barrier penetration is only significant when the classical phase-space area escaping through the cantori per period greatly exceeds Planck's constant. Experimental data and a quantum analysis confirm that the cantori act as barriers. We also briefly discuss the effect of decoherence in this system.

Investigations into chaotic systems have contributed much to the understanding of dynamics in both the classical and quantum domains. Classical phase space can contain the rich structure of resonances, Kolmogorov–Arnol'd–Moser (KAM) tori, cantori and regions of stochasticity. Knowing quantum mechanics to be the correct description of physical phenomena has motivated physicists to search for the signatures of these classical structures within the quantum regime. Hypersensitivity to initial conditions, a hallmark of classical chaos, is noticeably absent with quantum mechanics. The study [1] and experimental observation of *dynamical localization* [2, 3] dramatically displayed how quantum mechanics can eliminate stochastic diffusion. There has always been the hope that a clear transition between quantum and classical physics could be achieved, but whereas one can always theoretically allow $\hbar \longrightarrow 0$ and realize classical behavior, a small but nonzero $\hbar$ value always exists in the laboratory. With $\hbar \neq 0$ there is a clear distinction between the properties of classical chaos and its associated quantum counterpart.

The presence of KAM tori and cantori in the classical phase space are predicted to influence the corresponding quantum system. If a KAM boundary is unbroken it will prohibit classical diffusion through it, but quantum mechanical tunneling across the barrier is possible. When interaction terms in the perturbing Hamiltonian are sufficiently large so as to break up the boundary and create a cantori or *turnstile*, classical particles will quickly diffuse through the cantori but the quantum wavefunction will be inhibited [4–7]. Given a periodic perturbing Hamiltonian, a heuristic model proposes that when the classical phase-space area escaping through the cantori each period is $\sim\hbar$ then quantum diffusion is constrained [6, 7]. Even though the barrier has been broken, the quantum wavefunction still appears to *tunnel* through the cantori. The quantum system somehow senses the KAM boundary, resulting in a diminished probability for penetration of the barrier than that predicted classically.

By using laser cooled atoms within a modulated standing wave of light one can create a pristine environment for studying quantum chaos [2, 3, 8–10]. The observation of dynamic localization in the atomic optics realization of the δ-kicked rotor [8–10] provided an important experimental link to the most studied system in Hamiltonian chaos. In this present letter, we again subject our ultracold cesium atoms to a pulsed standing wave of near resonant light. A periodic pulsed potential of finite width (as used in the δ-kicked rotor experiments [8–10]) produces a KAM cantori, which becomes more noticeable for longer pulse widths. However, a train of single pulses is not the best system for observing diffusion through cantori, as the classical phase space outside the boundary is not strongly chaotic and contains many regular regions. We therefore subjected our atoms to a train of double pulses. The pulse train consists of two rectangular pulses of duration $\tau_p = 1.25$ μs with their leading edges separated by $\tau_s = 2.5$ μs. The double pulses occur at every $T = 25$ μs. Adopting the notation used in [8–10], we can write the dimensionless form of the Hamiltonian as

$$H = \frac{\rho^2}{2} - k\cos\phi\sum_{l=1}^{N} f(t-l), \qquad (1)$$

where $f(t)$ specifies the temporal shape of the pulses; $f(t) = 1$ for $0 < t < \tau_p$ and $\tau_s < t < \tau_s + \tau_p$ and zero otherwise. For an infinite train of kicks the Hamiltonian can be written as

$$H = \frac{\rho^2}{2} - k\sum_{r=-\infty}^{\infty} a_r\cos(\phi - 2\pi rt), \qquad (2)$$

where $a_0 = 1/10$ and $a_{r\neq 0} = [\sin(3r\pi/20) - \sin(r\pi/20)]/(r\pi)$. The classical interpretation of (2)

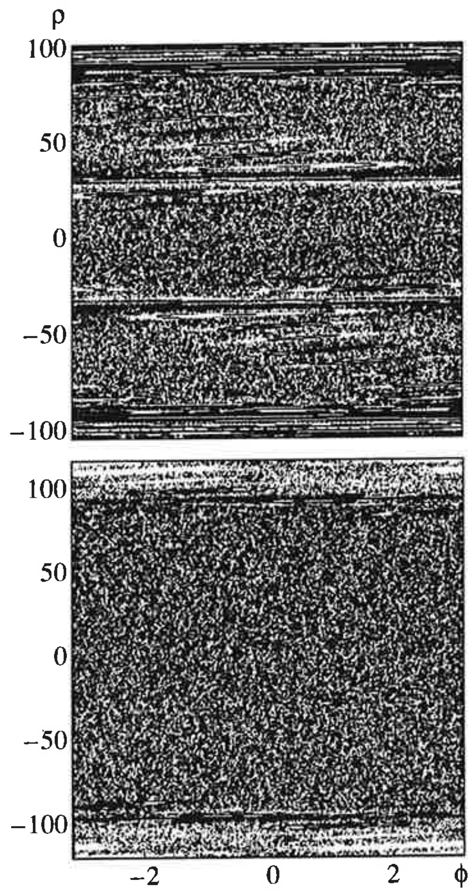

Fig. 1. Classical phase space for the relatively small kick strength $k = 70$ (top), with boundaries clearly visible at $\rho = 10\pi$ and $\rho = 30\pi$. For the kick strength of $k = 300$ (top) the $\rho = 10\pi$ cantori has completely broken up.

shows that the fundamental resonances are located at $\rho_r = 2\pi r$. The benefit of using this type of pulsed system is displayed when one considers the a_r terms. The KAM boundary, located at $\rho = 10\pi$, corresponds to the zero energy of the $r = 5$ term. Note that there is a relatively large amount of energy for $r > 5$. One can see in the Poincaré sections displayed in Fig. 1 that the $\rho = 10\pi$ cantori separates two strongly chaotic regions; the boundary is clearly visible for $k = 70$ but is completely broken up when $k = 300$. Another cantori at $\rho = 30\pi$ provides the upper boundary to the second region, and remains unbroken for all kick strengths used in our experiment. We prepare our atoms so that they initially lie within the $\rho = 10\pi$ cantori, and we monitor their subsequent evolution through the barrier.

In order to establish the connection between the dimensionless parameters used above and those that can be achieved in a laboratory, we consider an atom (transition frequency ω_0) suspended in a standing wave of near resonant light (frequency ω_l). Under the assumption of a large detuning compared to the Rabi frequency, the atoms' excited-state amplitude can adiabatically be eliminated. Then the resulting Hamiltonian governing the *coherent* time evolution reads [8–10]

$$H = \frac{p^2}{2m} - \frac{\hbar\Omega_{\text{eff}}}{8}\cos(2k_l x)\sum_{q=1}^{N} f(t - qT), \tag{3}$$

where $\Omega_{\text{eff}} = \Omega^2(s_{45}/\delta_{45} + s_{44}/\delta_{44} + s_{43}/\delta_{43})$ and $\Omega/2$ is the resonant Rabi frequency corresponding to a single beam. The terms in brackets take account of the different dipole transitions between the relevant hyperfine levels in cesium ($F = 4 \longrightarrow F' = 5, 4, 3$). The δ_{4j} are the corresponding detunings and, assuming equal populations of the Zeeman sublevels, the numerical values for the s_{4j} are $s_{45} = 11/27$, $s_{44} = 7/36$, $s_{43} = 7/108$. The function $f(t)$ represents the shape of our double kicks, discussed above. The dimensionless Hamiltonian is then recovered with $\phi = 2k_l x$, $\rho = 2k_l Tp/m$, $t' = t/T$, and $H' = (4k_l^2 T^2/m)H$; the primes are subsequently dropped. The kick strength is $k = \Omega_{\text{eff}}\omega_R T^2$, and $\omega_R = \hbar k_l^2/2m$ is the recoil frequency. The quantum features of the DKR enter through the commutation relation $[\phi, \rho] = i\bar{k}$, where $\bar{k} = 8\omega_R T$. The relationship between momenta is $n = p/(2\hbar k_L) = \rho/\bar{k}$.

Explicit details of the experimental setup can be found in [10]. Briefly, approximately 10^5 Cs atoms are initially trapped cooled in a magneto-optic trap (MOT) to a temperature of 10 μK; the position distribution of the trapped atoms has a FWHM of 200 μm. After trapping and cooling, the magnetic field gradient, the trapping beam and the repumping beam are turned off, leaving the atoms in the $F = 4$ ground state. The atoms are then subjected to a modulated periodic potential from a third laser diode. This beam passes through an acousto-optic modulator (AOM), is collimated to a measured waist ($1/e^2$ intensity radius) of 0.95 mm, and is then retroreflected from a mirror outside the vacuum cell to generate the one-dimensional potential. The maximum Rabi frequency at the MOT center is $\Omega/2\pi = 100$ MHz, for a total power of 10 mW. The finite widths of the kicking beam waist and the atomic cloud results in a narrow distribution of k with rms spread of 6% and $k_{\text{mean}} \approx 0.94k_{\text{max}}$ (k_{max} the value on the beam axis). In the following, when specifying k, this always refers to k_{mean}. The kick strength k was varied by adjusting Ω while maintaining a detuning of $\delta_{45} = 2.8$ GHz to the blue. For our detuning the spread in coupling strengths created by differing ac stark shifts of the different magnetic sublevels was below 1%. To measure the atomic momentum distribution we use a time-of-flight technique with a "freezing molasses" [8–10] with an expansion time of 12 ms.

The purpose of the experiment discussed in this letter is to examine the diffusion of particles through the KAM barrier at $\rho = 10\pi$. Our temperature of 10 μK places the initial momentum distribution within the

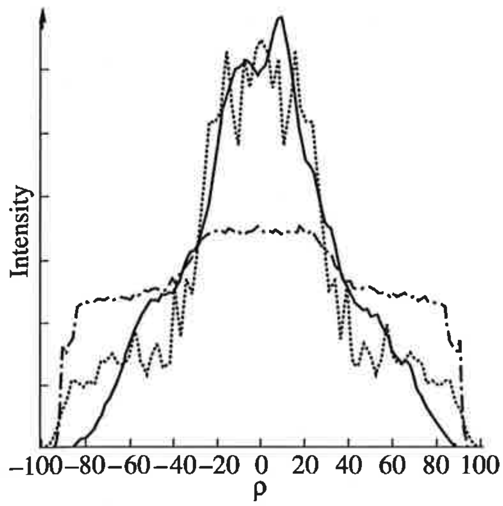

Fig. 2. An example of a measured momentum distribution (solid) for kick strength $k = 310$ and number of kicks $N = 56$, along with that predicted via a quantum (dotted) or classical (dot–dashed) analysis. The vertical lines correspond to the location of the barriers at $\rho = 10\pi$ and $\rho = 30\pi$.

cantori. After a number of kick cycles of sufficient strength, the resulting atomic momentum distribution quickly resembles that displayed in Fig. 2. The effects of the cantori are clearly discernible. For this example we had $k = 310$ and $N = 56$. Note the wings in the distribution outside of the $\rho = 10\pi$ cantori, but before the $\rho = 30\pi$ boundary. For comparison we display the predicted distributions for quantum and classical calculations. One can see the temporal development of the measured momentum lineshapes in Fig. 3 for $k = 310$. Here the number of kicks is increasing between 1 and 70. At large kick numbers, the system settles into an equilibrium configuration.

In keeping with the work of Geisel *et al.* [4, 5] we measured and calculated the percentage of particles that would cross the $\rho = 10\pi$ cantori as a function of kick number and kick strength. All our measurements were at a fixed “Planck’s constant” of $\bar{k} = 2.6$. In this present work we also purposely avoid the influence of spontaneous emission. The study of how decoherence introduced via spontaneous emission can affect the

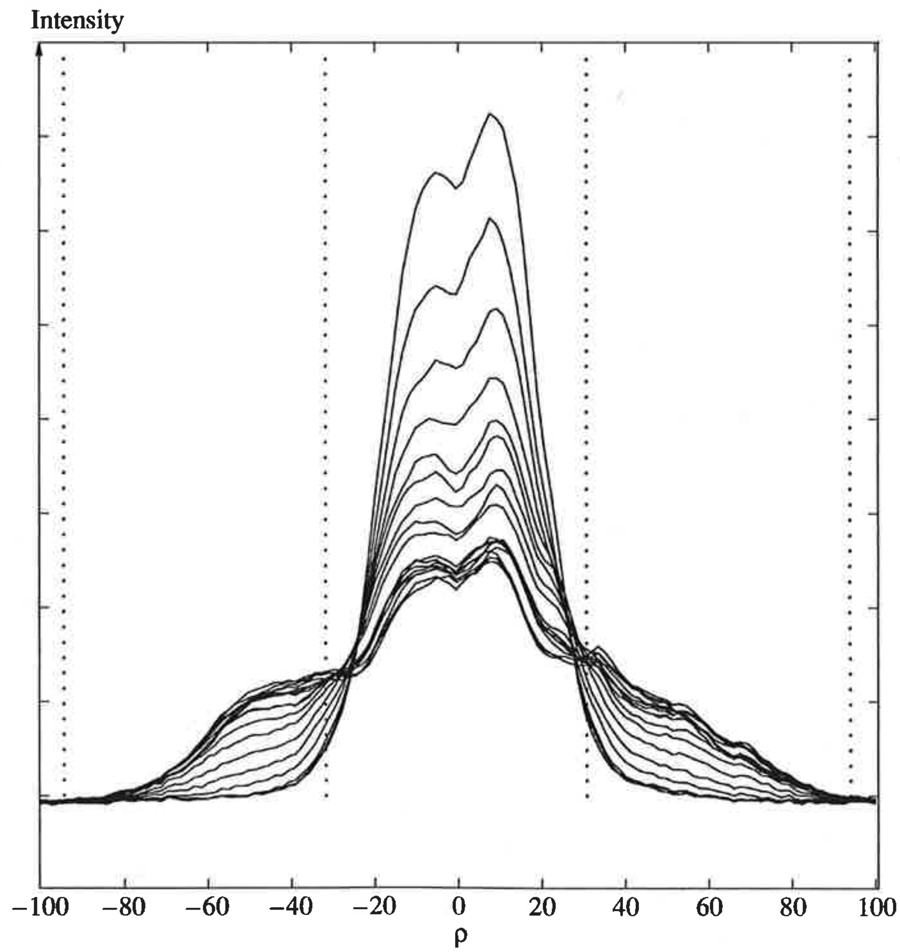

Fig. 3. Measured momentum distribution for kick strength $k = 310$ as the number of kicks increases (N = 1, 2, 4, 7, 12, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70). The vertical lines correspond to the location of the barriers at $\rho = 10\pi$ and $\rho = 30\pi$.

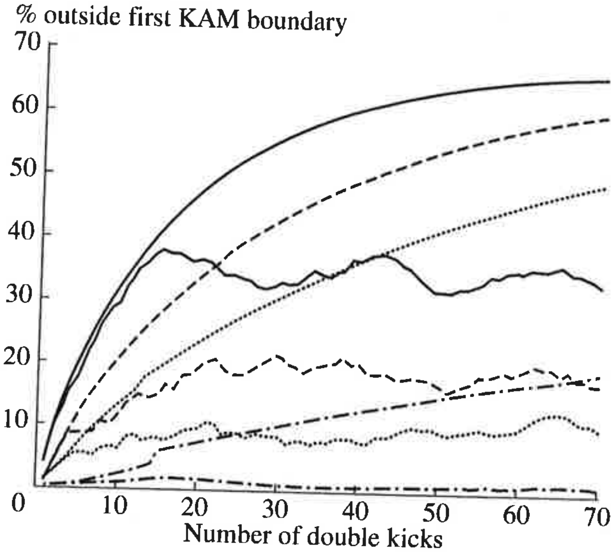

Fig. 4. The probability for finding the particles outside the $\rho = 10\pi$ cantori, versus kick number as calculated via classical (upper) and quantum (lower) analyses for kick strengths of $k = 345$ (solid line), 280 (dashed line), 220 (dotted line), and 140 (dot–dashed line).

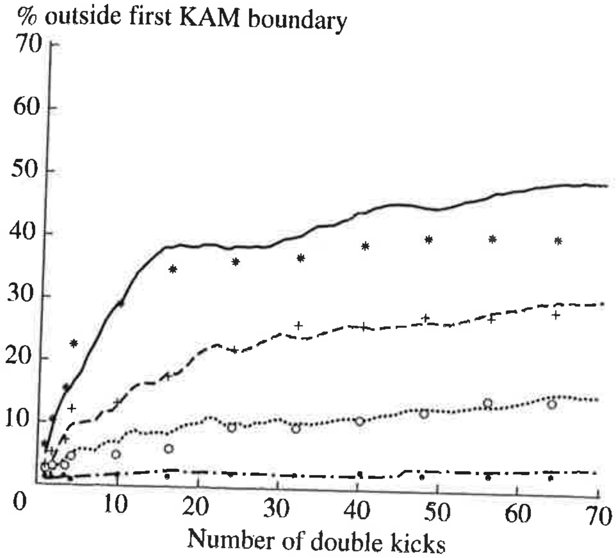

Fig. 5. The probability for finding the particles outside the $\rho = 10\pi$ cantori, versus kick number for experimental (data points) and a quantum simulation (lines) that includes spontaneous emission for kick strengths of $k = 345$ (* and solid line), 280 (+ and dashed line), 220 (○ and dotted line), and 140 (● and dot–dashed line).

ability of particles to flow through the cantori is intriguing, and will be presented in a forthcoming publication. In this present study we accounted for spontaneous emission by including it in our quantum computation, and comparing it to a computation where the effect is absent. We can therefore confirm that our experimental results are from a regime where the influence of spontaneous emission is minimal, but still noticeable. Our quantum analysis is based on a density matrix calculation. Spontaneous emission is included via the inclusion of an interaction term, $H_{\rm int} = -\zeta u \bar{k} \phi \sum_{l=1}^{N} \delta(t-l)$ to the Hamiltonian (1), where ζ is either 0 or 1, $\langle\zeta\rangle = \eta$ = probability for spontaneous emission per double kick, and $u\bar{k}$ is the recoil momentum projected onto the kicking beam axis (u chosen randomly on the interval [−1, 1]).

The presence of KAM cantori and their influence on the momentum distribution of atoms in similar experiments has been previously observed [3]. The Austin group recently carried out a systematic investigation of the finite pulse rotor system [11], with single kicks. By increasing the duration of the pulses, they bring in the KAM boundary and observe a reduction in the energy of the atomic distribution.

There exists a heuristic explanation for the inhibition of quantum diffusion through partial barriers and turnstiles [6, 7]. When the flux per period of phase space area escaping through classical cantori is comparable to or less than Planck's constant, then quantum diffusion is suppressed and penetration through the barrier is only via tunneling. This hypothesis is supported by our experimental observations and theoretical analysis. In Fig. 4 we display the results of quantum and classical analyses that predict the percentage of atoms that will be found outside the $\rho = 10\pi$ cantori as a function of kick number for various kick strengths; both the classical and quantum calculations omit spontaneous emission. The initial distribution corresponds to 10 μK, or 99.9% of the atoms within the $\rho = 10\pi$ cantori. Our computer simulations reveal that the KAM boundary is broken at $k \leq 50$. With the barrier broken the classical particles will eventually spread themselves uniformly between the $\rho = \pm 30\pi$ barriers, thereby giving a probability to be found outside the $\rho = 10\pi$ cantori of 2/3. For k = 280, 310, and 345 the classical particles reach this equilibrium in about 100, 70, and 50 kicks, respectively.

We have numerically calculated the classical phase space flux through the cantori per kicking cycle. We assume the particles are uniformly spatially distributed, but with initial momentum of $\rho = 10\pi$. Referring to the kick strengths displayed in Fig. 4, we find that the phase space area per period moving across the cantori to larger momenta is proportional to k^2, with values of $6.7\,\bar{k}$, $4.5\,\bar{k}$, $2.9\,\bar{k}$, and $1.2\,\bar{k}$ for k = 345, 280, 220, and 140, respectively. The ability of the cantori to constrain the quantum particles is clearly displayed. The quantum system quickly reaches its own equilibrium, with significantly fewer atoms outside the cantori. Only for kick strengths around k = 1200 does the quantum system begin to mimic the classical, with the probability for penetration of the cantori exceeding 60% and the phase space area per period moving across the cantori at 31 $\bar{k}$.

The experimental results for our diffusion experiment are displayed in Fig. 5. The measured probabilities of finding our Cs atoms outside the $\rho = 10\pi$ cantori are displayed. Also shown are the results of a quantum analysis that includes the spontaneous emission rates for the experimental parameters. The probability per kicking cycle of spontaneous emission for the traces are $\eta = 2.4$, 2.0, 1.6, and 0.8% for $k = 345$, 280, 220, and 140, respectively. We see good agreement between the measured and calculated probabilities. For strong kick strengths the final measured probabilities are noticeably smaller than those predicted classically; for $N \approx 65$ we measure 40, 30 and 15% for $k = 345$, 280, and 220 respectively, whereas the classical prediction gives 66, 58, and 43%. The broken cantori still dramatically restricts the movement of atoms through the classically broken barrier. By blocking our retroreflected beam we experimentally confirm that our small spontaneous emission rates create minimal heating and do not produce any movement of the atoms through the cantori.

The resolution of our CCD (19 pixels/mm), coupled with our 12 ms expansion time, allowed us to determine the position of the $\rho = 10\pi$ momentum line to an accuracy of $\Delta\rho = \pm 0.8$. At our strongest kick strength of $k = 345$ this results in an uncertainty in our measured probability of +/– 4%. We repeat our experiment a number of times during each experimental session, and the resulting spread in our measured values is +/– 4%.

Different process can contribute to a quantum system's inhibition to diffusion. Dynamic localization is certainly the reason for the elimination of momentum diffusion in the kicked rotor [1] and its atomic optics realization [8–10] at sufficiently high kick strengths; we note that others [12] disagree with this conclusion for the experiment descried in [2, 3]. Recent theoretical work on discontinuous quantum systems displays dynamic localization and cantori inhibition of diffusion in the same system [13]. For the experiment described in this present paper the cantori acts as barriers. Whereas dynamic localization results in an exponential lineshape for the momentum distribution in the kicked rotor, the quantum version of our double kick system quickly settles into the type of distribution that is displayed in Fig. 2. Our measurements and quantum analysis show that the system's probability for penetration of the KAM barrier saturates at values far below the classical prediction.

The decohering effect of spontaneous emission is small but still noticeable in our data. Whereas a fully quantum system saturates and the percentage of atoms found beyond the $\rho = 10\pi$ cantori remains constant, spontaneous emission creates a slow drift in the system toward the classical equilibrium configuration where 2/3 of the particles are outside this cantori. The coupling of a quantum system to extraneous degrees of freedom, or the *environment*, destroys quantum coherences. In these quantum chaos experiments utilizing atomic optics the quantum dynamics become susceptible to the decohering effects of spontaneous emission. The environment in this case is the vacuum fluctuations. A heuristic model for the decoherence observed in our double kick system, along with further experimental data, will be presented in a forthcoming publication. The model is similar to that presented in [9, 10]. However, from an experimental view of our present data one can clearly observe the slow drift of our quantum system towards the classical equilibrium configuration.

With a periodic train of double (finite width) pulses we create a system with two strongly chaotic regions separated by a KAM boundary. Using laser-cooled atoms and a pulsed laser beam, we have a pristine environment for experimentally observing a quantum system's inhibition to diffusion through cantori. Previous theoretical work [4–7] supports the hypothesis that when the flux of classical phase space through turnstiles is less than Planck's constant the movement of particles across the barrier is restricted. Our measurements and calculations of the movement of atoms through the cantori supports the conclusion that when $\bar{k} \approx 1$ the system fails to see the holes in the cantori [13]. The restriction of the quantum particles momentum is remarkable when one considers the underlying classical phase space displayed in Fig. 1. For the kick strengths used in our experiment the $\rho = 10\pi$ cantori is not noticeable in the Poincaré section, just an apparent region of stochasticity constrained by the barrier at $\rho = 30\pi$. However, the effect of the cantori remains significant for the quantum system.

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