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Experimental observation of dynamical localization and decoherence in the atomic δ-kicked rotor

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Abstract. We experimentally investigate decoherence by spontaneous emission in the quantum delta kicked rotor. Caesium atoms in a magneto-optic trap are initially cooled, and then subjected to a pulsed standing wave of near resonant light. As the detuning from the atomic transition is varied, we observe a change in the momentum diffusion after the quantum break time.

1. Introduction

The mechanism of decoherence plays a significant role in contemporary quantum physics, because it has the potential to solve the long-standing problems in measurement theory. However controversial, the most promising attempts to explain the act of measurement from within quantum mechanics rely on environment-induced decoherence [1] or some sort of coarse graining [2]. Furthermore, a coupling to extraneous degrees of freedom is an essential element in various, more experimentally motivated approaches using stochastic Schrödinger equations; an example of this kind is the quantum-state diffusion model [3], which mimics individual quantum systems rather than tracking a whole ensemble.

It has long been thought that, at least for all *practical* purposes, macroscopic superposition states only arise when quantum measurements are performed. However, after the discovery [4] of exponentially unstable wavefunctions in quantum systems with a chaotic classical limit, this picture had to be revised. Chaotic semiclassical systems develop phase space structures of order $\hbar$ in logarithmically short times $\sim \ln(1/\hbar)$, which can be comparable to typical dynamical times. This poses questions pertaining to the quantum–classical correspondence principle. However, as correspondence is closely connected to quantum measurements, it is expected that the ultimate answer will involve decoherence. This is why it is interesting to study open quantum systems which are chaotic in their classical limit.

The standard map, which describes the dynamics of the classical delta kicked rotor (DKR), is indisputably the most studied system in Hamiltonian chaos. For an ensemble of classical particles, a chaotic phase space entails diffusion in momentum space. A similar diffusion is observed in the quantum DKR up to the break time, but then dynamical localization sets in [5]. This effect has recently been observed by Moore *et al* [6], who experimentally realized the DKR by exposing a cloud of ultracold sodium atoms to a one-dimensional optical potential which was pulsed on periodically in time. Incoherent photon

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scattering processes were negligible due to a large detuning between the laser light and the atomic transition.

In this paper we report on the experimental observation of decoherence in the open quantum DKR. Dynamical localization is a coherent effect and as such it will be influenced by a coupling of the system to its environment [7–10]. In the atom optics realization of the DKR, the dynamics becomes susceptible to decoherence by spontaneous emission. An important experimental feature is the fact that its rate, and thus the coupling strength between the atoms and the vacuum fluctuations (the environment), can be controlled by varying the detuning between the atomic transition and the optical field frequency. Measurements of the momentum distribution of atoms in the pulsed lattice, as a function of the detuning, thus provide a direct examination of the loss of coherence in a quantum system.

2. DKR model

In order to establish the connection between the experimental parameters and those appearing in the well known DKR model, we consider an atom (transition frequency ω_0) suspended in a standing wave of near resonant light (frequency ω_l). We will see later that in order to destroy dynamical localization one does not have to introduce a large amount of spontaneous emission. Therefore, the atomic dynamics will be predominantly coherent and we consequently neglect spontaneous emission for the moment. Under the further assumption of a large detuning compared with the Rabi frequency, the atoms' excited-state amplitude can adiabatically be eliminated. Then the resulting Hamiltonian governing the *coherent* time evolution reads [6]

$$H = \frac{p^2}{2m} - \frac{\hbar\Omega_{\text{eff}}}{8}\cos(2k_l x)\sum_{q=1}^{N} f(t-qT), \tag{1}$$

where $\Omega_{\text{eff}} = \Omega^2(s_{45}/\delta_{45} + s_{44}/\delta_{44} + s_{43}/\delta_{43})$ and $\Omega/2$ is the resonant Rabi frequency corresponding to a single beam. The terms in brackets take account of the different dipole transitions between the relevant hyperfine levels in caesium ($F = 4 \rightarrow F' = 5, 4, 3$). The δ_{4j} are the corresponding detunings and, assuming equal populations of the Zeeman sublevels, the numerical values for the s_{4j} are $s_{45} = \frac{11}{27}$, $s_{44} = \frac{7}{36}$, $s_{43} = \frac{7}{108}$. The sum in the Hamiltonian (1) describes the temporal modulation of the potential. The function $f(t)$ represents the shape of the kicks, which in this work is close to rectangular: $f(t) = 1$ for $0 < t < \tau_p$ and zero otherwise. The pulses repeat with period T, and in the limit where τ_p becomes smaller they can be approximated by delta functions. The dimensionless DKR Hamiltonian is then recovered as

$$H = \rho^2/2 - \kappa\cos\phi\sum_{q=1}^{N}\delta(t-q) \tag{2}$$

where $\phi = 2k_l x$, $\rho = 2k_l T p/m$, $t' = t/T$ and $H' = (4k_l^2T^2/m)H$; the primes are subsequently dropped. The classical stochasticity parameter is $\kappa = \Omega_{\text{eff}}\omega_R T\tau_p$, and $\omega_R = \hbar k_l^2/2m$ is the recoil frequency. The quantum features of the DKR enter through the commutation relation $[\phi, \rho] = \mathrm{i}\bar{k}$, where $\bar{k} = 8\omega_R T$.

3. Experimental details

Our experimental set-up is similar to that of Moore *et al* [6]. Caesium atoms are initially trapped and cooled in a standard magneto-optic trap (MOT) within a glass cell of internal

pressure of 10^{-10} Torr. Anti-Helmholtz coils yield a magnetic field gradient of up to 15 G cm^{-1}. Two laser diodes at 852 nm are frequency stabilized via saturated absorption spectroscopy [11], and each beam is passed through a 70–90 MHz variable acousto-optic modulator (AOM). The trapping beam is held at approximately 20 MHz to the red of the $(6S_{1/2}, F = 4)$ to $(6P_{3/2}, F' = 5)$ transition, while the repumping beam is locked to the $(6S_{1/2}, F = 3)$ to $(6P_{3/2}, F' = 4)$ transition to prevent the accumulation of atoms in the $F = 3$ ground state. Each beam is spatially filtered by passing it through a single-mode optical fibre. The collimated outputs are divided into three pairs of $\sigma^+ \Leftrightarrow \sigma^-$ counterpropagating beams. Approximately 10^5 caesium atoms are initially captured in the MOT. A 20 ms cooling phase reduces the atomic gas' temperature to as low as 5 μK by further detuning (another 10 MHz) and decreasing the intensity of the trapping beam [12]. After this cooling phase, the position distribution of the trapped atoms has a FWHM of approximately 200 μm.

The modulated periodic potential is produced by a third laser diode. The beam passes through an 80 MHz AOM and a single-mode optical fibre. The collimated beam with a measured waist of $2\sigma = 1$ mm is then retro-reflected from a mirror outside the vacuum cell to generate the one-dimensional potential. Note that the finite width of the beam waist together with the non-zero width of the atomic cloud entails a (reasonably narrow) distribution of κ values with $\kappa_{\text{mean}} \approx 0.9\kappa_{\text{max}}$, where κ_{max} is the kicking strength on the beam axis. In the following, when specifying κ, this always refers to κ_{mean}. Taking reflection losses at the windows of the containing glass cell into account, the Rabi frequency in the centre of the MOT is $\Omega/2 = 2\pi \cdot 154$ MHz (rms). The temporal modulation is achieved via the radio frequency supply to the AOM; 40 ns is observed as the rise and fall time for the optical pulses. The pulse spacing used is $T = 20$ μs ($\bar{k} = 2.1$). The optical detuning is to the blue of the $F = 4$ to $F' = 5$ transition and is monitored by overlapping fractions of the kicking and trapping beams and measuring the beating frequency using a fast photodiode and a spectrum analyser. Note that both the Rabi frequency $\Omega/2$ and the pulse spacing T are held constant throughout this work. Varied are the pulse width ($\tau_p = 90$–500 ns) and the detuning ($\delta_{45}/2\pi = 0.62$–4.0 GHz) with the constraint that the classical stochasticity parameter remains constant at $\kappa = 12.5$. Therefore, only the spontaneous emission rate will change.

After trapping and cooling, the trapping beam is turned off leaving the atoms in the $F = 4$ ground state. They have at most a 1:6 chance per spontaneous scattering to fall into the $F = 3$ ground state, in which they would experience a much weaker potential. As this would considerably disturb the measurements, we leave the repumping beam on throughout the experiment. This produces a small additional heating, which however is of no importance as for our parameters heating effects are negligible. To measure the momentum distribution of the atoms we use a time-of-flight technique as follows. After subjecting the atoms to the pulsed periodic potential we let the cloud expand for 10 ms. Then the trapping laser is switched on, while the magnetic field gradient remains off. The atoms are frozen in place within a few ms by the created optical molasses, and the cloud is imaged on a CCD camera. The atomic motion during the CCD's integration time (some tens of ms) is negligible compared with the spatial extent of the cloud. The experimental sequence and data acquisition are computer controlled.

Note that the rectangular shape of the kicking pulses causes KAM boundaries in classical phase space even in the case of large κ values. This is because the pulse frequency spectrum has zeros at integer multiples of $2\pi/\tau_p$. For the longest pulses used in this work $\tau_p = 500$ ns, the first boundary appears at a dimensionless momentum of $n \equiv p/2\hbar k_l \approx 120$. We have observed this regular region, so it was easy to make sure that it did not affect the

measurements. We also mention that we operated well away from quantum resonances, the first of which (a half-resonance [6]) we experimentally observe at $T = \pi/4\omega_R = 61\ \mu$s.

4. Experimental results

For sufficiently low spontaneous emission rates, the atomic momentum distribution initially diffuses, followed by the onset of dynamical localization (an important experimental observation reported by Moore *et al* [6]). Figure 1(*a*) displays the CCD measured image of the trapped caesium atoms before the kicking commences, while figure 1(*b*) depicts the frozen cloud after a series of 30 kick cycles *and* the subsequent free expansion (figure 1(*b*) therefore corresponds to a momentum distribution, as opposed to figure 1(*a*)). The elliptic shape is due to the interaction with the pulsed lattice. It is interesting to note that by blocking the retro-reflected beam, thus turning the standing wave into a travelling wave, the effect disappears. This confirms that we are indeed dealing with coherent two-photon processes and, moreover, that heating effects are negligible. In figure 2 the image of the cloud is projected onto one dimension (i.e. integrated over the direction perpendicular to the beam axis). One can clearly see the signature of dynamical localization. The characteristic exponential distribution $\sim \exp(-2|n|/l_s)$ has a localization length of $l_s \approx 19$. This agrees well with the theoretical prediction $l_s \approx 2\xi \approx \kappa^2/2\bar{k}^2 \approx 18$, where ξ is the localization length of the participating Floquet states.

Spontaneous emission introduces decoherence to the DKR [7–10]. This destroys dynamical localization and results in *quantum diffusion*, namely the momentum diffusion *after* the quantum break time. Figure 3 displays the measured growth of the atoms' kinetic energy for different detunings. The initial diffusion rate is held constant by choosing smaller pulse widths for smaller detunings. For the three displayed traces, the probabilities for spontaneous emission per kick are $\eta = 0.76 \times 10^{-2}$, 2.3×10^{-2} and 4.6×10^{-2}, respectively. The initial 'classical-like' diffusion can clearly be distinguished from the quantum diffusion. Although 0.76×10^{-2} seems to be a small scattering probability, it is apparent that there is considerable quantum diffusion even in this case of large atom–laser detuning ($\delta_{45}/2\pi = 4.0$ GHz). This reflects the vulnerability of quantum coherences, which is expected to be enhanced by the underlying classical chaoticity. It should be mentioned that we could not increase the detuning any further while maintaining a high chaoticity because of the limited power provided by the kicking beam laser diode. It is perhaps worth noting that we observe exponentially localized momentum distributions not only for the situation displayed in figure 2, but also in the case of small detunings. This is somewhat surprising considering the high quantum diffusion rate. However, based on an analytic calculation, a similar behaviour has been found in the case of a phase-modulated potential [10], where it has been shown that the shape becomes Gaussian only for atomic momenta much larger than the localization length.

In order to try to understand the atoms' kinetic energy growth for the different scattering probabilities η, we heuristically derive an analytical expression for the quantum diffusion coefficient $\mathcal{D}_\infty \equiv \lim_{N\to\infty}\langle n^2\rangle/N$ as follows. Let us assume that one spontaneous scattering event causes complete decoherence between the atomic wavefunction and the Floquet states [10]. Realizing that the measured diffusion at a given instant will be a mixture of contributions from different atoms at different stages of their time evolution, the diffusion coefficient can be written as $\mathcal{D}_\infty = \sum_{k=0}^{\infty} \eta(1-\eta)^k D(k)$, where $D(k)$ is the time-dependent diffusion coefficient in the absence of spontaneous emission. Using $D(k) = D_0 \exp(-k/N^*)$ [8], we arrive at $\mathcal{D}_\infty = \eta N^* D_0/(1+\eta N^*)$, where the parameters D_0 and N^* denote the initial diffusion coefficient and the crossover time, respectively ($N^* \gg 1$ has been used).

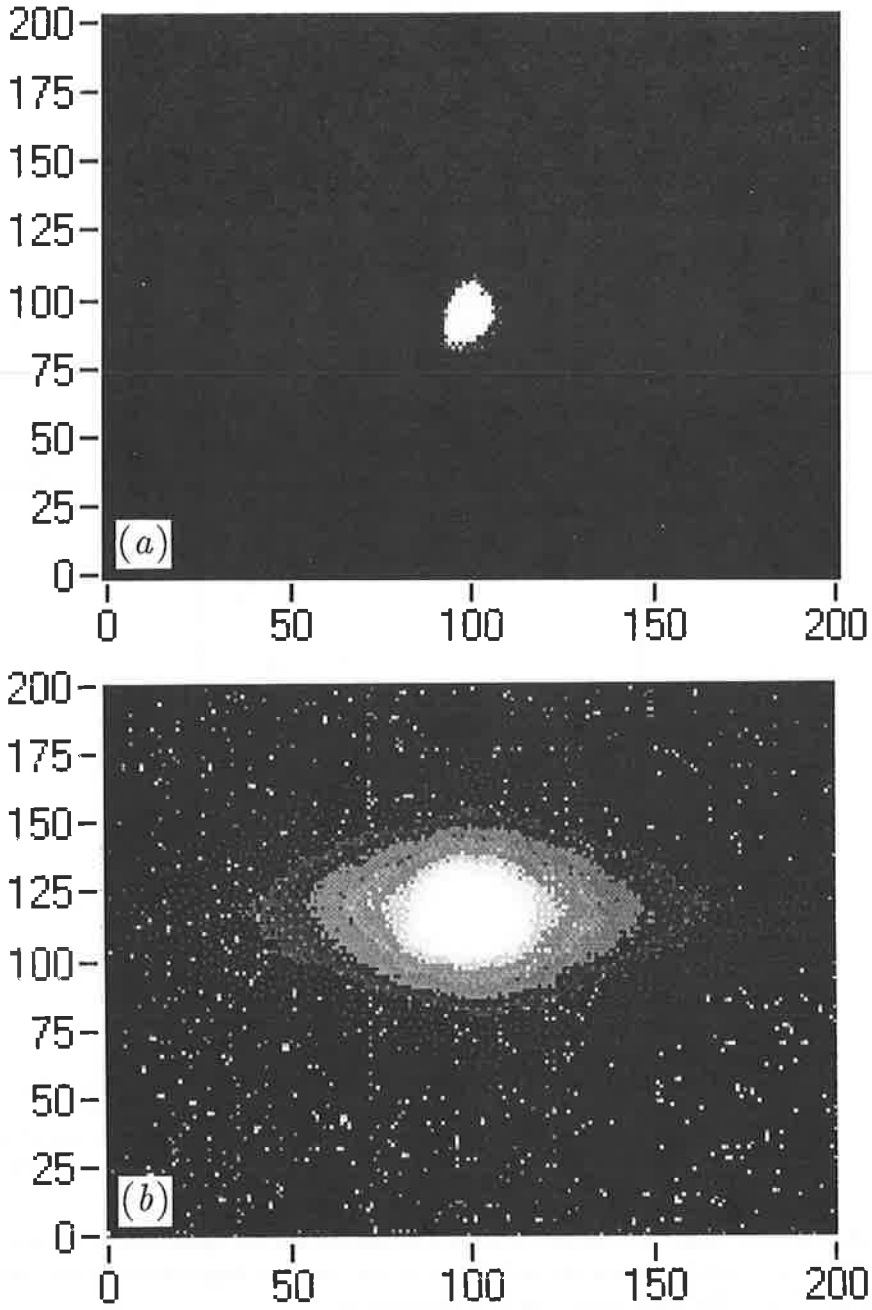

Figure 1. A display of the CCD measured image of the trapped atoms (a) before the kicking commences and (b) the cloud in the molasses after a series of 30 kicks and subsequent free expansion (10 ms). The kicking beam axis is horizontal, and the experimental parameters are $\kappa = 12.5$, $\bar{k} = 2.1$ and $\delta_{45}/2\pi = 4.0$ GHz. The numbers denote pixels; the scaling is 19 pixels mm^{-1} (horizontal) and 26 pixels mm^{-1} (vertical).

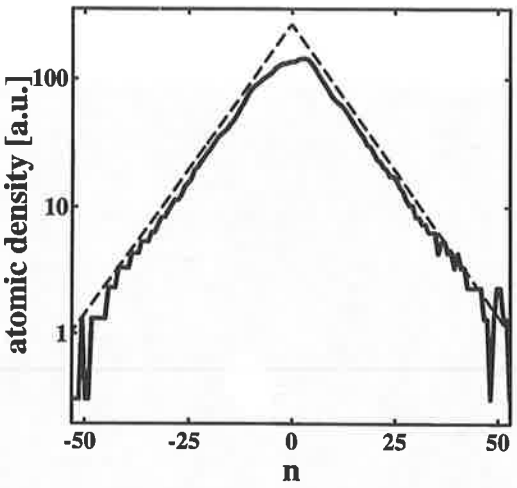

Figure 2. The exponentially localized momentum ($n \equiv p/2\hbar k_L$) distribution after 30 kicks on a logarithmic scale. The parameters are the same as in figure 1. The localization length corresponding to the broken curve is $l_s = 19$.

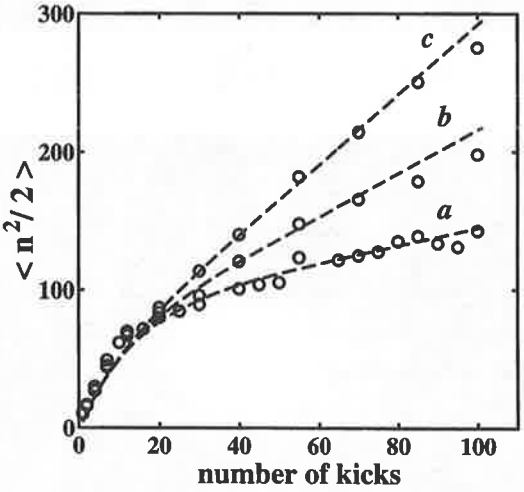

Figure 3. Kinetic energy $\langle n^2/2 \rangle$ as a function of number of kicks N. The experimental (circles) results are for $\kappa = 12.5$, $\bar{k} = 2.1$ and (*a*) $\eta = 0.76 \times 10^{-2}$, (*b*) $\eta = 2.3 \times 10^{-2}$, (*c*) $\eta = 4.6 \times 10^{-2}$. The corresponding detunings are $\delta_{45}/2\pi = 4$ GHz, 1.3 GHz and 0.62 GHz, respectively. The parameters for the analytical (broken) traces are $D_0 = 13$ and $N^* = 14$ (η values as above).

Along similar lines, one can derive an expression for the *time-dependent* diffusion coefficient and, by a summation over the number of kicks, the time dependence of the kinetic energy $E \equiv \langle n^2/2 \rangle$. The final result can be expressed as

$$E(N) = \mathcal{D}_\infty N/2 + [(D_0 - \mathcal{D}_\infty)/2] \cdot [(1 - q^N)/(1 - q)], \tag{3}$$

where $q \equiv (1 - \eta)\exp(-1/N^*)$. The energy growth described by this heuristic formula is displayed in figure 3. We find consistency between the measured data (each point being the average of 25 individual runs) and the analytical expressions for $D_0 \approx 13$ and $N^* \approx 14$. These values imply a saturation energy in the absence of spontaneous emission of $E_{\text{sat}} \equiv$

$\langle n^2/2\rangle_{\rm sat} = D_0 N^*/2 \approx 90$, which is consistent with the experimental data. Note that the approximate formula for the diffusion coefficient $D_0 \approx \kappa^2/2\bar{k}^2$ yields $D_0 \approx 17.7$, which is somewhat larger than our measured $D_0 \approx 13$. The measured energies have an estimated 20% uncertainty, due primarily to the fact that most of the atom's kinetic energy is contained in the wings of the momentum distribution, where the signal-to-noise ratio of the CCD is small.

We plan to continue the study of decoherence via spontaneous emission in the atomic DKR. Especially intriguing in this context is the quantum–classical correspondence principle. We have seen that the quantum diffusion rate tends towards the classical rate with an increasing degree of decoherence. In the literature, this is often referred to as 'driving the system back to the classical behaviour'. This notion certainly seems to be sensible when considering the remarks in the introduction, according to which decoherence is considered to be a necessary condition for quantum–classical correspondence (in the limit $\bar{k} \to 0$). Whether it is also a sufficient condition in the system studied here ($\bar{k}$ = constant), i.e. whether a larger quantum diffusion rate implies a higher degree of classicality, requires further studies and will be addressed in a forthcoming publication.

5. Conclusion

Using laser-cooled caesium atoms we have observed dynamical localization in the atom optics realization of the DKR. This builds on the previous observation of the effect with sodium atoms [6]. The introduction of spontaneous emission destroys localization which gives rise to a kinetic energy growth after the quantum break time. The exponentially shaped momentum distribution characteristic for dynamical localization is observed even for 'large' spontaneous emission rates up to 4.6% per kick. We have demonstrated that the quantum diffusion rate tends towards the classical rate with an increasing degree of decoherence.

Acknowledgments

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