

Review

Stochastic gravitational wave backgrounds

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Abstract

A stochastic background of gravitational waves could be created by the superposition of a large number of independent sources. The physical processes occurring at the earliest moments of the universe certainly created a stochastic background that exists, at some level, today. This is analogous to the cosmic microwave background, which is an electromagnetic record of the early universe. The recent observations of gravitational waves by the Advanced LIGO and Advanced Virgo detectors imply that there is also a stochastic background that has been created by binary black hole and binary neutron star mergers over the history of the universe. Whether the stochastic background is observed directly, or upper limits placed on it in specific frequency bands, important astrophysical and cosmological statements about it can be made. This review will summarize the current state of research of the stochastic background, from the sources of these gravitational waves to the current methods used to observe them.

Keywords: gravitational waves, cosmology, astrophysics

(Some figures may appear in colour only in the online journal)

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1. Introduction

Gravitational waves are a prediction of Albert Einstein from 1916 [1, 2], a consequence of general relativity [3]. Just as an accelerated electric charge will create electromagnetic waves (light), accelerating mass will create gravitational waves. And almost exactly a century after their prediction, gravitational waves were directly observed [4] for the first time by Advanced LIGO [5, 6]. The existence of gravitational waves had already been firmly established in 1982 through the observation of the orbital decay of a binary neutron star system [7]; as the two neutron stars orbited around one another they were accelerating, so gravitational waves were emitted, carrying away energy and causing the orbit to decay. Advanced LIGO has subsequently observed gravitational wave signals from merging binary black hole systems [8–11]. Since then Advanced LIGO and Advanced Virgo [12] have joined together to observe a binary black hole merger [13] and a binary neutron star merger [14]. The detection of gravitational waves from the binary neutron star merger, GW170817, marked the beginning of gravitational wave multi-messenger astronomy, with simultaneous observations of the event and its source across the electromagnetic spectrum [15]. It can be argued that multi-messenger astronomy started with the joint electromagnetic and neutrino observations of SN 1987A [16].

A gravitational wave is a traveling gravitational field. An electromagnetic wave is a traveling electric field and magnetic field, both transverse to the direction of propagation. Similarly, the effects of a gravitational wave are transverse to the direction of propagation. The effects of a gravitational wave are similar to a tidal gravitational field. In terms of general relativity, a gravitational wave will stretch one dimension of space while contracting the other. Just like electromagnetic waves, gravitational waves carry energy and momentum with them.

Gravitational waves are far too weak to be created by some process on the Earth and then subsequently detected. Energetic astrophysical events will be the source of observable gravitational wave signals. The events could be the inspiral of binary systems involving black holes or neutron stars. Core collapse supernovae could produce a detectable signal if they occurred in our galaxy, or perhaps in nearby galaxies. A spinning neutron star would produce a periodic gravitational wave signal if the neutron star had an asymmetry that made it nonaxisymmetric. Finally, there could be a stochastic background of gravitational waves made by the superposition of numerous incoherent sources. The recent detection by LIGO and Virgo of gravitational waves from the coalescence of binary black hole and binary neutron star systems implies that there is a stochastic background created by these sorts of events happening throughout the history of the universe [9, 17, 18]. Because of the recent LIGO–Virgo results there will be an emphasis in this report on the stochastic background that LIGO–Virgo may soon observe; however, searches via other methods will also be addressed. Certainly different processes in the early universe have created gravitational waves. For example, quantum fluctuations during inflation [19], the speculated period of exponential growth of the universe at its earliest moments, have created gravitational waves that would be observed as a stochastic background today [20].

This report will give an overview of the stochastic gravitational wave background (or, more simply, in this report, the stochastic background). Presented will be a summary of the various means by which a stochastic background could be created. Furthermore, the different ways that a stochastic background could be detected will be presented, along with the information that could be extracted from its observation, or even the absence of its observation.

1.1. Gravitational waves

Given here is a brief review of gravitational waves. For a comprehensive summary of gravitational wave physics, sources, and detection methods, see [21–24]. Working with linearized general relativity, the gravitational wave is assumed to make only a slight modification to flat space,

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}, \quad (1)$$

where $g_{\mu\nu}$ is the spacetime metric, $\eta_{\mu\nu}$ in the Minkowski metric (representing flat spacetime), and $h_{\mu\nu}$ is the metric perturbation. The generation of gravitational waves is a consequence of general relativity, and can be predicted via the Einstein equation. To first order in the metric perturbations, gravitational waves are created when the mass quadrupole moment is accelerating, namely it has a non-zero second derivative with respect to time. Gravitational waves also carry energy and momentum. When a system emits gravitational waves, it loses energy. The existence of gravitational waves was first confirmed through the observation of the orbital decay of the binary pulsar PSR 1913 + 16 [7, 25]; the rate at which the orbit for this system is decaying exactly matches the prediction from general relativity for the loss of energy through gravitational wave emission. This is also the reason

for the coalescence of the binary black holes and the binary neutron stars observed by Advanced LIGO and Advanced Virgo, such as GW150914 [4], GW151226 [8], GW170104 [10], GW170608 [11], GW170814 [13] and GW170817 [14].

After emission, a gravitational wave essentially travels as a plane wave. Imagine a wave traveling in the z -direction. Just as with electromagnetic radiation, there are two possible polarizations, and the physical effects are transverse to the direction of propagation. We can arbitrarily choose our x and y axes. One polarization, which we will call the $+$ polarization, will cause space to be expanded and contracted along these x and y axes. The other polarization, which we will call the \times polarization, will cause space to be expanded and contracted along the x' and y' axes, where these axes are rotated by 45° from the other axes.

Let us look in detail at the effect of the $+$ polarization. Consider the plane wave moving in the z -direction

$$h_{ij}(z, t) = h_+ \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} e^{i(kz - \omega t)}. \quad (2)$$

Spacetime is stretched due to the strain created by the gravitational wave. Starting with a length L_0 along the x -axis, the gravitational wave causes the length to oscillate as

$$L(t) = L_0 + \frac{h_+ L_0}{2} \cos(\omega t). \quad (3)$$

There is a change in its length of

$$\Delta L_x = \frac{h_+ L_0}{2} \cos(\omega t). \quad (4)$$

Along the y -axis, a similar length L_0 subjected to the same gravitational wave oscillates as

$$\Delta L_y = -\frac{h_+ L_0}{2} \cos(\omega t). \quad (5)$$

In this example, the x -axis stretches while the y -axis contracts, and then vice versa, as the wave propagates through the region of space. In terms of the relative change of the lengths of the two arms (at $t = 0$),

$$\Delta L = \Delta L_x - \Delta L_y = h_+ L_0 \cos(\omega t), \quad (6)$$

or

$$h_+ = \frac{\Delta L}{L_0}. \quad (7)$$

The amplitude of a gravitational wave, h_+ , is the amount of strain that it produces on spacetime. The other gravitational wave polarization (h_\times) produces a similar strain on axes 45° from (x, y) . The stretching and contracting of space is the physical effect of a gravitational wave, and detectors of gravitational waves are designed to measure this strain on space.

1.2. Sources of gravitational waves

When searching for gravitational waves the signals are roughly divided into four categories: coalescing binaries, unmodeled bursts (for example from core collapse supernovae),

continuous waves (for example from pulsars), and stochastic. The signal search techniques are then optimized for these particular signals.

Compact binary coalescence will produce a typical chirp-like signal. In the LIGO–Virgo observational band, from 10 Hz up to a few kHz, these signals will be made from binary systems consisting of neutron stars (with masses $\sim 1.4 M_\odot$) and black holes (with masses up to $\sim 100 M_\odot$). As the binary system's orbit decays via energy loss by gravitational wave emission, the two objects spiral into one another. The orbital frequency increases, and consequently the gravitational wave frequency and amplitude also increase. In addition to the inspiral (chirp) signal, there will also be a signal associated with the merger of the two objects, and if a black hole is created, the ringdown signal as the black hole approaches an axisymmetric form. Since the binary inspiral signal is relatively straightforward to calculate, the LIGO–Virgo signal search is based on comparing the data with templates. As the ability to predict the form of the signal has improved, these templates now account for the spin of the masses [26–28]. Once the signals are detected, Bayesian parameter estimation routines are used to extract the physical parameters of the system. These methods now incorporate the full extent of the waveform: inspiral, the merger of the two masses, and the black hole's ringdown to an axisymmetric form [29]. It is interesting to note that stellar mass binary black hole systems, similar to GW150914 [4], will also be visible in the proposed space-based gravitational wave detector [30], the laser interferometer space antenna (LISA) [31, 32]. LISA will be able to observe these systems weeks to years before they coalesce in the LIGO–Virgo band. LISA will observe gravitational waves with frequencies between 0.1 mHz and 100 mHz. In this band LISA will also observe binary black hole systems with masses up to $\sim 10^7 M_\odot$. Pulsar timing methods (whereby the regular radio signals from pulsars are used like clocks in the sky, and the presence of a gravitational wave would vary the arrival time of the pulses) will search for supermassive binary black hole systems, with masses from $3 \times 10^7 M_\odot$ to $3 \times 10^9 M_\odot$ (gravitational wave periods of the order of years) [33].

There are several possible sources of unmodeled bursts of gravitational waves. Core collapse supernovae are one of the most exciting possibilities. The gravitational wave emissions from these sorts of events are extremely difficult to predict [34]. Other burst signals could come, for example, from pulsar glitches, or the transition of a neutron star to a black hole. These types of signals are typically searched for via excess power in the data. LIGO and Virgo have recently searched for signals of these types with durations from a few milliseconds up to 10 s [35]. The inspiral and merger of a very massive binary black hole pair will be of short duration in the LIGO–Virgo observation band, so these excess power detection methods will be the most effective means of observing them. There are also different mechanisms by which there could be gravitational wave transients of significant amplitude for extended periods; LIGO and Virgo are currently looking for burst events lasting up to 1000 s [36, 37]. In addition to excess power types of searches, it is also possible to search for cosmic string signals via a dedicated template based search [38].

Cosmic strings are theorized to be one-dimensional topological defects created after a spontaneous symmetry phase transition [39, 40] as predicted in a range of field theories. While cosmic string kinks and cusps will produce short duration transient gravitational wave signals, the forms of these signals are technically predictable.

Neutron stars are extremely dense, and often spinning at incredible rates. It is suspected that neutron stars typically have masses around $1.4 M_{\odot}$, with a radius around 12 km. Neutron stars can have significant angular velocities; there is evidence of a pulsar with a rotation rate of 714 Hz [41]. A rotating sphere will not emit gravitational waves (due to conservation of mass); more generally, a rotating axisymmetric object will not emit gravitational waves (due to conservation of angular momentum). However, if there is some asymmetry in the shape of the rotating neutron star, then it can emit gravitational waves. These gravitational waves would be periodic, but due to other factors (loss of energy from gravitational wave emission, or accretion from a companion in a binary), there can be a frequency derivative. The Doppler shift between the source and the detector must also be considered. With these factors in mind, LIGO and Virgo are currently searching for gravitational waves from rapidly rotating neutron stars [42, 43]

The incoherent sum of numerous unresolved gravitational wave signals will result in a stochastic background of gravitational waves. This is the main topic of this report, and much more information on this background is presented below.

The magnitude of the stochastic gravitational wave background is usually reported in terms of its energy density per logarithmic frequency interval with respect to the closure density of the universe ($\rho_c = \frac{3c^2 H_0^2}{8\pi G} \approx 7.6 \times 10^{-9}$ erg cm⁻³ with $H_0 = 67.74$ km s⁻¹ Mpc⁻¹, $h = 0.6774$ [44], c the speed of light and G Newton's constant), or specifically

$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df}. \quad (8)$$

One can also consider the energy density of gravitational waves over a particular frequency band, namely $\Omega_{\text{GW}} = \int d \ln f \Omega_{\text{GW}}(f)$ [45]. The stochastic gravitational wave background could come from cosmological sources: the inflationary epoch, phase transitions in the early universe, alternative cosmologies, or cosmic strings. Alternatively, there could be an astrophysically produced cosmological background. This could be produced from supernovae, magnetars, or the inspiral and merger of compact objects (neutron stars or black holes) over the history of the universe. Because of the recent observation of stellar mass binary black hole and binary neutron star mergers by Advanced LIGO and Advanced Virgo, it is likely that the stochastic background in the LIGO–Virgo observation band will be dominated by this source, with $\Omega_{\text{GW}}(f) \approx 10^{-9}$ at 25 Hz [17, 18].

1.3. Summary of recent gravitational wave detections

In the first observing run of Advanced LIGO (O1, September 12, 2015–January 19, 2016) three gravitational wave signals

were observed. GW150914 was reported as a definitive gravitational wave observation, with the signal created by the merger of a binary black hole pair with masses $36 M_{\odot}$ and $29 M_{\odot}$, at a distance of 410 Mpc. The total energy emitted in gravitational waves was $3 M_{\odot} c^2$ [4]. The second definitive gravitational wave observation was GW151226. This event was the result of the merger of two black holes with masses of $14 M_{\odot}$ and $7.5 M_{\odot}$, at a distance of 440 Mpc. A total of $1 M_{\odot} c^2$ of energy was released as gravitational waves [8]. Finally, event LVT151012 was almost certainly a gravitational wave event, but because of the long distance to the source, 1000 Mpc, it had a reduced gravitational wave amplitude and signal-to-noise ratio, and hence a lower statistical significance. The masses for this system were $23 M_{\odot}$ and $13 M_{\odot}$. The energy released in gravitational waves was $1.5 M_{\odot} c^2$ [9].

The second observing run (O2, November 30, 2016–August 25, 2017) of Advanced LIGO and Advanced Virgo has provided more events. Advanced Virgo joined O2 on August 1, 2017. Advanced LIGO observed gravitational waves from binary black hole mergers GW170104 (with masses of $19.4 M_{\odot}$ and $31.2 M_{\odot}$ at a distance of 880 Mpc) [10] and GW170608 (with masses of $12 M_{\odot}$ and $7 M_{\odot}$, the lightest binary black hole system observed to date, at a distance of 340 Mpc) [11]. The first three-detector observation of gravitational waves between Advanced LIGO and Advanced Virgo was the detection of GW170814, another binary black hole system (with masses of $25.3 M_{\odot}$ and $30.5 M_{\odot}$ at a distance of 540 Mpc) [13]. The Advanced LIGO and Advanced Virgo network then detected gravitational waves from a binary neutron star inspiral, GW170817 [14]; a gamma ray burst was detected 1.7 s after the merger [46–48], and the source was identified across the electromagnetic spectrum [15], thus beginning the era of gravitational wave multi-messenger astronomy.

1.4. What is a stochastic gravitational wave background?

A stochastic background of gravitational waves is very different from transient gravitational waves (binary inspirals, or burst events) or continuous periodic gravitational waves (coming from pulsars). These other sources are sending gravitational waves from specific locations in the sky. A stochastic background will come from all directions. To a first approximation, the stochastic background is assumed to be isotropic; one could determine its statistical properties by observing any part of the sky [49, 50]. Searches for the stochastic background typically proceed with the hypothesis that it is uniform across the sky [51]. This is analogous to the cosmic microwave background (CMB), which is essentially isotropic, but, in fact it is ultimately anisotropic (with temperature anisotropies at the level of 10^{-5}) [52, 53]. Similarly, there are signal searches that attempt to measure an anisotropic stochastic gravitational wave background [54].

Unlike other gravitational wave signals, a stochastic background would just appear as *noise* in a single gravitational wave detector. For example, consider some detector attempting to measure gravitational waves. The signal $s(t)$ from that

detector would be the sum of the gravitational wave, $h(t)$, and noise, $n(t)$, or specifically,

$$s(t) = n(t) + h(t). \quad (9)$$

However, the magnitude of a stochastic background will always be much smaller than the noise in the detector, $n(t) \gg h(t)$. The only way to detect a stochastic background will be to take the correlation between two detector outputs,

$$\begin{aligned} \langle s_1(t) s_2(t) \rangle &= \langle (n_1(t) + h(t)) (n_2(t) + h(t)) \rangle \\ &= \langle n_1(t) n_2(t) \rangle + \langle n_1(t) h(t) \rangle + \langle h(t) n_2(t) \rangle \\ &\quad + \langle h(t) h(t) \rangle \\ &\approx \langle h(t) h(t) \rangle, \end{aligned} \quad (10)$$

(where the $\langle \rangle$ represents the time average) since it is assumed that the noise in each detector is statistically independent from one another, and also from the stochastic background.

In reality, the two detectors will be displaced from one another, so the detected signal will not be quite the same; the consequences of this will be articulated below. Also, having two co-located detectors typically leads to common noise, as was the case for initial LIGO when it used two co-located detectors to attempt to measure the stochastic background [55]; Advanced LIGO does not have co-located detectors. As a consequence, LIGO and Virgo are attempting to measure the stochastic background through the correlation of the output of detectors displaced thousands of kilometers from one another. The assumption was that there would be no common noise, but even this assumption cannot be sustained [56–58].

As will be described below, numerous different methods will be used to try to measure a stochastic background in different frequency regimes. In all likelihood, the stochastic background's energy level will change very little over the observational band of the detector. There will not be large variations in the background when looking at it in the frequency domain, nor in the time domain. The stochastic background would essentially be impossible to detect in a single detector. But through the correlation of data from different detectors one could possibly extract the signal. In terms of formal statistical definitions, it is assumed that the background is stochastic, stationary, and ergodic [59].

There is certainly a stochastic gravitational wave background at some level. From all of the activity over the history of the universe, space-time is constantly oscillating. Using the stochastic background to probe the earliest moments of the universe, for example from inflation [19], would provide an unprecedented window to the physics of the early universe [20, 60]. The gravitational waves produced in the early universe will have frequencies today that extend from $1/T_{\text{Hubble}}$ to at least 10^{14} Hz, if not higher [49, 50]. However, for LIGO and Virgo, their observational band (from 10 Hz to a few kHz) is likely to be dominated by a stochastic background produced by the merger of binary black holes and binary neutron stars over the history of the universe [17, 18].

A properly calibrated gravitational wave detector will produce an output of the measured gravitational wave strain, $h(t)$ (which is dimensionless). From the correlation of the output

of two detectors one can measure the root mean square (rms) of the strain, h_{rms}^2 , or the spectral density $S_h(f)$,

$$h_{\text{rms}}^2 = \left\langle \sum_{ij} h_{ij} h_{ij} \right\rangle = \int_0^\infty df S_h(f). \quad (11)$$

The energy density of the gravitational waves can be related to the spectral density, namely

$$\rho_{\text{GW}} = \int_0^\infty df \rho_{\text{GW}}(f) = \int_0^\infty df S_h(f) \frac{\pi c^2 f^2}{8G}, \quad (12)$$

with

$$\frac{d\rho_{\text{GW}}}{df} = \rho_{\text{GW}}(f). \quad (13)$$

In this case, equation (8) can be written as

$$\Omega_{\text{GW}}(f) = \frac{f \rho_{\text{GW}}(f)}{\rho_c}. \quad (14)$$

1.5. The importance of observing a stochastic gravitational wave background

Whether produced by cosmological or astrophysical sources, an observed stochastic gravitational wave background would provide a wealth of information about this universe. This is analogous to the CMB; the observation of it and its anisotropies has revolutionized our understanding of the universe [44, 52, 53, 61]. An even deeper view of the universe could come from the stochastic gravitational wave background. Gravitational waves from inflation would help describe the universe at its earliest moments [62–69]. There is also the possibility that the initial state of the universe was perturbed via string cosmology. With string cosmology there could be a phase of accelerated evolution in advance of the Big Bang. This would also create a distinctive background of gravitational waves [70–73]. These pre-Big-Bang cosmologies might produce gravitational waves that could be observed in the LIGO–Virgo observational band [70–73]. Cosmic strings, theorized topological defects produced by phase transitions in the early universe, vibrate and lose energy via gravitational wave emission over the history of the universe [39, 74–76]. If cosmic strings exist, they will create a stochastic background of gravitational waves, the observation of which would bring confirmation of physics beyond the Standard Model [77]. A first-order phase transition in the early universe would see the production of bubbles of different phases. The growth of spherical bubbles would not create gravitational waves, but the collision of bubbles would. The observation of a stochastic background produced by first-order phase transitions would certainly provide significant information on cosmology and high-energy physics [78–80].

An astrophysically produced stochastic gravitational wave background certainly exists at some level. The recent observations by Advanced LIGO and Advanced Virgo of binary black hole and binary neutron star mergers [4, 8–11, 13, 14] imply that a stochastic background will be produced by these events happening over the full history of the universe [17, 18, 51].

A stochastic background produced by binary black hole mergers is likely to be the loudest background in the LIGO–Virgo band, and one that may ultimately be observable by those detectors [18]. The merger of binary neutron star systems over the course of the universe will also contribute significantly to the stochastic background [18, 81–83]. An astrophysically produced stochastic background would have contributions from core collapse supernovae [84, 85], rotating neutron stars [86], differentially rotating neutron stars [87], and magnetars [88] throughout the universe. Any information derived from an astrophysically produced stochastic background would provide significant information about astrophysical processes over the history of the universe. Clearly the differentiation between the different sources of a stochastic background will be difficult to observe and will ultimately require observation of the frequency dependence of the stochastic background over an extended frequency band.

1.6. Methods used to measure a stochastic background

There are many methods that are currently being used to try to observe the stochastic background of gravitational waves. A number of techniques have been proposed for future attempts to observe the stochastic background. These methods will be reviewed below. However, a recent review provides an extremely comprehensive explanation of all of the methods used and proposed to observe the stochastic background, and the interested reader is encouraged to consult that summary [24]. In addition the article [45] provides an excellent overview on observational limits on the stochastic background over 29 decades in frequency.

LIGO and Virgo have used correlation methods between two or more interferometric detectors to attempt to measure the stochastic background [49, 50, 89]. While no signal was detected, upper limits have been placed on the energy density of the background from 20 Hz to 1000 Hz [51, 55, 90–95]. Pulsar timing has been used to try to detect a stochastic background in the 10^{-9} Hz– 10^{-8} Hz band [45]. The temperature and polarization anisotropies of the CMB can be used to constrain the energy density of the stochastic gravitational wave background in the 10^{-20} Hz– 10^{-16} Hz band [45, 96]. The normal modes of oscillation of the Earth can even be used to constrain the stochastic background energy density in the 0.3 mHz–5 mHz band [97].

In the future (probable launch in the 2030s), the space based gravitational wave detector LISA [31] will search for a stochastic background in the 0.1 mHz–100 mHz band. Earth based atomic interferometers are being proposed to search for gravitational waves, including a stochastic background, in the 0.3 Hz–3 Hz band [98]. A detector such as this would occupy an important location in the frequency spectrum between LISA and LIGO–Virgo. The proposed, space-based DECI-hertz Interferometer Gravitational wave Observatory (DECIGO) would attempt to observe gravitational waves from 0.1 Hz to 10 Hz [99, 100].

Presented in section 3 will be a more detailed description of the methods to observe the stochastic background, what

their sensitivities are at present, and what their sensitivities are expected to be in the future.

2. Summary of sources of a possibly observable stochastic gravitational wave background

There are a number of sources of stochastic backgrounds. Below we summarize the most probable backgrounds produced via cosmological or astrophysical phenomena. An excellent review of astrophysically produced stochastic backgrounds can be found in [101]; however, the implications of the observations by Advanced LIGO and Advanced Virgo of gravitational waves from binary black holes and binary neutron stars has significantly increased the probability that an astrophysically produced stochastic background will be observed in the near future [17, 18].

2.1. Inflation

The electromagnetic analog to the stochastic background is the cosmic microwave background (CMB). In the early universe the fundamental particles and photons were in thermal equilibrium. Up until about 400 000 yr after the Big Bang, protons, electrons and photons formed a cosmic soup, and continuously bounced off one another. However, due to the expansion of the universe the temperature of the universe dropped, and neutral hydrogen was eventually formed. This event is referred to as *recombination*, although it is the first time in which electrons and protons combined to form neutral hydrogen. At this moment the photons were free to propagate away, and essentially did not interact anymore with matter.

The CMB was observed for the first time, albeit accidentally, in 1964 (when the age of the universe was 13.8 billion years [44, 53, 102]) by Arno Penzias and Robert Wilson of Bell Laboratories, in New Jersey, USA [103]. The explanation of the cosmological origin of the observation was published simultaneously [104], although the existence of the CMB had been predicted before [105]. The CMB is observed today to have a perfect black body temperature distribution corresponding to 2.726 K [106, 107]. There are slight temperature anisotropies across the sky of the order of 30 μ K rms [108]. From these temperature fluctuations, specifically how they vary as a function of angular scale, it is possible to estimate the cosmological parameters that describe our universe [44, 53, 102, 109].

While the cosmological information provided by the CMB is astounding, specific features of the CMB raise a number of questions. For example, any two points on the sky separated by more than 2° were causally disconnected at the time of recombination. This then leads to the question: how is it possible that the temperature of two points on opposite sides of the sky have the same temperature (to one part in 10^5) if they have not been in thermal equilibrium with each other? This is what is known as the *Horizon Problem*. The temperature fluctuations of the CMB as a function of angular scale on the sky can be used as input for Bayesian parameter estimation methods [102, 109] that then allow for the estimation of cosmological

parameters [44, 53]. From this, as well as other methods, it is apparent that the present energy density of the universe (considering radiation, baryonic matter, dark matter, dark energy) seems to be equal, or nearly equal, to the closure density of the universe

$$\rho_c = \frac{3H_0^2}{8\pi G} = 7.8 \times 10^{-9} \text{ ergs cm}^{-3} \quad (15)$$

with a Hubble constant of $H_0 = 67.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [44]. If the current energy density of the universe is equal to the critical energy density then the curvature of the universe is zero, namely the universe is flat. The question then becomes, how is it possible that we find ourselves in such a special state of curvature? And if we are just close to a curvature of zero now, then earlier in the universe the curvature must have been even closer to zero. This is what is known as the *Flatness Problem*.

The theory of *inflation* solves these problems [19, 110]. It is assumed that in the very earliest moments the universe went through a period where its size grew exponentially, namely $a(t) \propto e^{H_{\text{vac}} t}$, where $a(t)$ is the scale parameter of the universe, and H_{vac} is the Hubble parameter at that time [111]. This expansion could be caused by the presence of some scalar field, let us call it ϕ , which would give the space at that time some energy density, ρ_{vac} , which would then be related to the square of the Hubble parameter by $H_{\text{vac}}^2 \propto \rho_{\text{vac}}$ [111]. Eventually the decay of the scalar field to our present vacuum would put an end to the exponential inflation at that time, and provide the energy for the production of the fundamental particles that we are aware of today.

This rapid expansion of the universe has the effect of driving the curvature of the universe to zero, thus solving the Flatness Problem. It also means that our entire observable universe occupied a region which was in causal contact, and presumably thermal equilibrium, before the effect of the exponential expansion drove the regions apart from one another.

At this early period in the universe quantum mechanics would have played an important role in the evolution of the universe. All quantum fields have vacuum fluctuations associated with them. This would have been true for the inflationary field ϕ as well. Scalar fluctuations in the field could have served as the initial seeds for the distributions of matter that we see in the universe today. However, there would also have been tensor fluctuations, and these would have produced gravitational waves [65–68, 112–115]. Gravitational waves could also be produced at the end of inflation, during the period of *pre-heating*, when the scalar field was decaying into the material that makes up the present day universe [64, 69]. These primordial gravitational waves, if observed, could provide information about the universe in this inflationary era.

The gravitational waves produced during inflation would exist today over wavelengths corresponding to the size of the observable universe, down to sub-atomic distances. For frequencies above 10^{-17} Hz the predicted background is around $\Omega_{\text{GW}} \approx 10^{-15}$, a level that will likely be difficult to observe by any technique at any wavelength. Note that for lower frequencies (10^{-17} Hz corresponds to a period of 23% of the age of the universe) there is an increase in the predicted energy density of the stochastic background as perturbations from the

early universe that were *frozen out* (being larger than observable size of the universe) re-enter and propagate again as gravitational waves. Of course, alternative inflationary scenarios could produce a stochastic background at different levels.

2.2. Cosmic strings

Cosmic strings are a unique possibility for new physics that could be observed via gravitational waves. These would be one-dimensional topological defects, or false vacuum remnants, produced after a spontaneous symmetry phase transition [39, 40] from a broad variety of field theories, for example, Grand Unified Theories applied in the early universe [116]. Their formation happens at the end of inflation [117].

Cosmic strings are classical objects. Cosmic superstrings are other theorized objects; these would be quantum objects, even though they would extend to cosmological distances [118]. The formation of cosmic superstrings would occur at the end of *brane* inflation, when D-branes annihilate, or via brane collisions [118].

When cosmic strings intersect they always swap partners, or when a single string folds upon itself, the connection interchange creates a cosmic string loop [119, 120]. On the other hand, when cosmic superstrings intersect the probability of swapping partners is less than one [118], even much less than one [121]. This can lead to an excess in the density of cosmic superstrings [121]. The intercommutation probability, p , is a very important parameter concerning the production of gravitational waves in the universe. There are predictions that the intercommutation probability p should be in the range of 10^{-1} –1 for D-strings, or 10^{-3} –1 for F-strings [122]. Cosmic strings and cosmic superstrings create gravitational waves [76]. When cosmic strings intersect, cusps and kinks will be formed. Cosmic string kinks [123–125] are discontinuities on the tangent vector of a string, while cusps are points where the string instantaneously reaches the speed of light [75, 76, 126]. These cusps and kinks will create bursts of gravitational waves, whose waveforms can be predicted [76, 126, 127]. The superposition of these gravitational waves from cosmic strings produced over the history of the universe will create a stochastic background of gravitational waves [75, 76].

Cosmic strings are characterized by the dimensionless tension of the string, $G\mu$ (assuming $c = 1$), where μ is the mass per unit length and G is Newton's constant. The product $G\mu$ is thus an unknown parameter that will affect the production of gravitational waves, and can be constrained by searches for gravitational waves (even null results) [38, 94, 128, 129].

Assuming that the magnitude of loops is defined by the gravitational backreaction scale (namely, the effect of the emitted gravitational waves changing the state of the cosmic string that created them), the null search results from initial LIGO place upper limits on the string tension of $G\mu < 10^{-8}$ for particular regions of the cosmic string parameter space [38]. The string tension has also been constrained through observations of the CMB to be less than 10^{-7} [128, 130–132]. Cosmic string loops will oscillate, producing gravitational waves [133, 134]. Combining gravitational wave observations [38] and cosmological data (CMB [135–137], baryon acoustic

oscillations [138–140], gravitational lensing data [141]), and again assuming that the size of the loops is determined by the gravitational backreaction scale, string tension values greater than 4×10^{-9} are excluded for an intercommutation probability of $p = 10^{-3}$ [128].

The data from Advanced LIGO and Advanced Virgo are now being used to search for cosmic string gravitational wave signals. The analysis of the Advanced LIGO data from the first observing run, O1, has recently been published [142]. No gravitational wave signals from cosmic strings were observed. That fact, along with the upper limits set on the energy density of the stochastic background from the Advanced LIGO O1 data, $\Omega_{\text{GW}} < 1.7 \times 10^{-7}$ for 20–86 Hz [51], was used to constrain three cosmic string models. One model (M1) assumes that all cosmic string loops were formed with roughly the same size, and the loops do not self-interact after they were created [40, 143, 144]. The next model (M2) uses numerical calculations to predict the size of the cosmic string loops when they were created, as well as the creation rate as a function of time [145]. The third model (M3) differs from M2 in that it considers the distribution (as a function of time) of loops that do not self-interact; it also considers the back-reaction on the loops when gravitational waves are emitted [146, 147]. The lack of detection of such gravitational wave bursts in the Advanced LIGO O1 data constrains M3, assuming an intercommutation probability of $p = 1$, to have a string tension $G\mu < 1 \times 10^{-9}$; the O1 burst search does not significantly constrain M1 and M2. The results of the Advanced LIGO O1 upper limits for the energy density of the stochastic background essentially exclude M3. For M1, the O1 stochastic search result constrains the string tension, assuming $p = 1$, to be $G\mu < 5 \times 10^{-8}$, for M2 the constraints are weaker, and with a reduction of intercommutation probability to $p = 0.1$, a tension constraint of $G\mu < 5 \times 10^{-8}$ can also be set. See [142] for the complete details of this study.

2.3. First-order phase transitions

In the physics world there are many types of phase transitions. In our day-to-day lives we see transitions between solid, liquid and gaseous matter. A particular medium in thermal equilibrium will have uniform characteristics pertaining to its physical qualities. But when a phase transition occurs, some of these physical characteristics will change. Some of the changes can even happen discontinuously [148].

A first-order phase transition has a discontinuity in the first derivative of the free energy with respect to a thermodynamic parameter. Consider the Gibbs free energy

$$G(p, T) = U + pV - TS, \quad (16)$$

where p is the pressure, T is the temperature, U is the internal energy of the system, V is the volume, and S is the entropy.

From the Maxwell relations we have $S = -\frac{\partial G}{\partial T}_p$ and $V = \frac{\partial G}{\partial p}_T$. If these quantities were discontinuous, then we would have a first-order phase transition. As a simple example, consider water changing from a liquid to a gas, namely the water is boiling. Both the entropy, S , and the volume, V ,

change abruptly when going from one phase to the other. In fact, the change in entropy can be related to the latent heat of the process, $L = T\Delta S$.

Second-order phase transitions have a discontinuity in the second derivative (with respect to thermodynamic parameters) of the free energy, while the first derivatives remain continuous. For example, second-order phase transitions are observed in superconductors, or the ferromagnetic phase transition in iron.

First-order phase transitions in the early universe could produce a significant stochastic background of gravitational waves. The boiling water analogy can be made, but now one can imagine bubbles of a different phase of the universe forming from within another older phase. The early universe certainly experienced a number of phase transitions. If one considers the Standard Model, there was presumably a grand unification period when the electromagnetic, weak and strong forces were all unified. As the universe cooled there would have been a transition to a phase where the electroweak force and the strong force were separated. Eventually an electroweak phase transition would see the separation of the electromagnetic force from the weak force. The standard electroweak phase transition is not a first-order phase transition, but slight modifications to the Standard Model could produce a first-order electroweak phase transition [149]. It is estimated that a *cross-over* between the unified electroweak phase and the subsequent broken phase would have happened at a temperature of $T_c = 159.5 \pm 1.5$ GeV [150]. However, if some modification to the Standard Model would have produced a first-order phase transition at this energy scale then there would be a stochastic background of gravitational waves peaking at a frequency of about 260 mHz [79]. What makes this so exciting is that this is within the observation band of LISA [31, 32]. This is one of the reasons why outside of the LHC experiments at CERN, LISA may offer the best prospects for acquiring high-energy physics information, and especially possible extensions to the Standard Model.

The Standard Model extensions to the electroweak phase transition, if they existed, would have important physical consequences. Electroweak baryogenesis could help to explain cosmic baryon asymmetry [151]. Electroweak baryogenesis pertains to mechanisms that would produce an asymmetry in baryon density during the electroweak phase transition, and could then possibly explain the observed abundance of matter over anti-matter (baryon asymmetry) in the universe. Electroweak baryogenesis also satisfies the famous Sakharov conditions [152]: the interactions occur out of thermal equilibrium; charge (C) and charge-parity (CP) symmetries are violated; there is a violation of baryon number. Electroweak baryogenesis provides an example of a first-order phase transition that could address baryon asymmetry and also produce gravitational waves in the early universe. In this modification to the electroweak theory, bubbles (of a new vacuum phase) would be created when the Higgs field transitions into the vacuum state where the electroweak symmetry is spontaneously broken. These bubbles would then expand. The C and CP violation would occur when particles present scatter off of the front of the expanding bubble walls. The C and CP

asymmetries occurring in front of the expanding bubble wall would produce baryon number violation, giving more baryons (matter) than antibaryons (antimatter) [151, 153].

In addition to the possible explanation for one of the great mysteries of the universe—why we have a surplus of matter over antimatter—we also have a mechanism that can create a significant background of gravitational waves. The characteristics of the gravitational waves produced by a first-order phase transition depend on the expansion speed of the bubble walls, the latent heat of the transition, and the rate at which bubbles of the new phase are created [153, 154].

With first-order phase transitions, gravitational waves are created via different physical mechanisms. An expanding bubble will be spherical, so will not produce gravitational waves; however, when bubble walls collide, there will be gravitational wave production. The plasma that is present can also experience shocks, and these discontinuities between regions of different plasma properties could also generate gravitational waves [154–157]. After the bubble collisions there will be sound waves in the plasma; these can create gravitational waves [154, 158, 159]. Because of the very large Reynolds number that would exist for this fluid, turbulent motion results; a large magnetic Reynolds number leads to an amplification of the magnetic fields created by the movement of charges during the phase transition [160]. Finally, magnetohydrodynamic turbulence can produce gravitational waves; the magnetic fields and turbulent motions can create stresses that are anisotropic. This can ultimately be an efficient way to convert magnetic energy to gravitational wave energy [154, 161, 162]. All of these processes would typically be present after a first-order phase transition. The amount of gravitational waves produced by these different effects would depend on the dynamics of the first-order phase transition. The sensitivity of LISA in detecting a stochastic background will be of the order of $\Omega_{\text{GW}} \sim 5 \times 10^{-13}$ at 10^{-3} Hz [163]. Many of the modifications to the electroweak phase transition, making it first-order, would create a stochastic gravitational wave background that could be detectable by LISA [154]. The possibility of detecting a stochastic background created by a first-order phase transition in the early universe is an amazing opportunity to observe new physics outside of the standard model.

2.4. Pre-Big-Bang models

Some pre-Big-Bang models are an extension of the standard inflationary cosmology. The theories consider the consequences for cosmology when some version of superstring theory is applied. As noted above, the stochastic background of gravitational waves generated via quantum fluctuations during inflation would result in an energy density that is essentially flat in frequency, and currently at a very small level, $\Omega_{\text{GW}} \sim 10^{-15}$. In pre-Big-Bang models the universe would begin with a string perturbative vacuum scenario [70, 71, 73, 164]. The universe materializes via a highly perturbative initial state before the Big Bang. In the standard inflationary scenario there would have been an initial singularity [165, 166]. Superstring theory allows for the assumption that there is no

singularity associated with the Big Bang, and hence it is logical to extend time to before the Big Bang.

With string cosmology (namely the pre-Big-Bang scenario) there will be a different behavior for the curvature scale of the universe, as opposed to that in the standard inflationary cosmology. Standard inflation has a constant curvature scale before reaching the radiation-dominated (standard Big Bang) era. However, with string cosmology there would be a growth in the curvature scale, going from a low curvature scale to some maximum curvature scale that would be defined by the string scale. This is the so-called string inflation. The curvature scale of the universe would then diminish, and the radiation-dominated era of the standard cosmology would ensue. The universe would not have experienced a singularity with an infinite curvature scale, but would instead be finite through the effects of the stringy phase [164].

This dynamical process in the early universe would be a source of gravitational waves, and would create a stochastic background that would be present today [72, 164, 167]. Initially the universe would be in a low energy, low curvature-scale, *dilaton* phase. The dilaton is the assumed fundamental scalar for the string theory. An inflationary evolution would occur due to the kinetic energy of the dilaton field. As the curvature scale increases the universe is described by a high-energy string phase. Eventually the curvature scale approaches the string scale and higher order corrections become important in the string action. This is when the universe transitions to the radiation-dominated era described by the standard cosmology. These transitions from different expansion rates for the universe will create gravitational waves [72, 164, 167]. This process can then create a background that can peak at higher frequencies, possibly within the observation band of LIGO–Virgo or LISA. The parameters pertaining to the string phase will affect the frequency dependence of the stochastic background [72, 167]. Whether or not LIGO and Virgo will be able to observe a stochastic background from a pre-Big-Bang cosmology has been the subject of active investigation [72, 73].

Various observations already constrain pre-Big-Bang models. No stochastic background has been detected at this point, so the upper limits on the energy density of the stochastic background in various frequency bands can generate some restrictions on pre-Big-Bang theories. Specifically, observations of the CMB, and stochastic background energy limits set by Advanced LIGO and pulsar timing are currently able to constrain pre-Big-Bang parameters [45, 73, 128, 168].

This string cosmology would produce both scalar and tensor perturbations to the metric of the universe. Observations of the CMB, for example, from Planck, estimate cosmological parameters such that it appears that scalar perturbations are creating a stochastic background that is decreasing with frequency, in contrast to pre-Big-Bang predictions [44]. This constrains the parameters responsible for the very low frequency gravitational waves produced in the pre-Big-Bang evolution [73].

Pulsar timing arrays provide another important limit on the stochastic background that constrains pre-Big-Bang models [45]. For example, the Parkes Pulsar Timing Array

placed a limit on the energy density of the stochastic background of $\Omega_{\text{GW}}(f) < 2.3 \times 10^{-10}$ at $f = 1 \text{ yr}^{-1}$ [45, 169]. Finally, the recent upper limit from Advanced LIGO, $\Omega_{\text{GW}}(f) < 1.7 \times 10^{-7}$ from 20–86 Hz further constrains pre-Big-Bang models [51]. In order for the pre-Big-Bang models to exist within these observational constraints, fine tuning must be done on the string parameters. That said, it has still been demonstrated that pre-Big-Bang models could produce a stochastic background that peaks within the Advanced LIGO–Advanced Virgo observational band, or the LISA observational band [73].

2.5. Binary black holes

A stochastic background produced by binary black holes is highly probable. After Advanced LIGO’s observations of two significant events, and another probable event, in its first observing run (O1) it has become clear that there is likely to be a stochastic background produced by all binary black hole mergers over the history of the universe [4]. More binary black hole inspiral gravitational wave events were subsequently observed by Advanced LIGO and Advanced Virgo in the second observing run (O2) [10, 11, 13]. This astrophysically produced background will likely be the loudest stochastic background in the observation band of LIGO and Virgo, from 10 Hz up to 1000 Hz.

Immediately after the observation of GW150914 [4] LIGO and Virgo reported on the implications that the observation of a stellar mass binary black hole merger would have on the stochastic background [17]. The detection made it clear that the universe contains a population of stellar mass binary black holes. Consequently the binary black hole produced stochastic background should be larger than what was expected previously. This stochastic background would be created from all of the binary black hole mergers in the observable universe over its 13.8 billion year history. Using various scenarios and parameters for the formation of stellar mass binary black hole systems, LIGO and Virgo used the observation of GW150914 to predict that around 25 Hz (where Advanced LIGO and Advanced Virgo will have the best sensitivity in detecting a stochastic background) the estimated energy of the binary black hole produced stochastic background will be $\Omega_{\text{GW}}(f = 25 \text{ Hz}) = 1.1_{-0.9}^{+2.7} \times 10^{-9}$ [17].

LIGO and Virgo have now observed several binary black hole mergers. In O1 there were GW150914 [4], GW151226 [8] and the probable (but not definitive) LVT151012 [9]. At the time of this writing, LIGO and Virgo have announced the detection of three binary black hole mergers observed in their second observing run, O2: GW170104 [10], GW170608 [11], and GW170814 [13]. Using these observations, LIGO and Virgo now estimate that the energy of the binary black hole produced stochastic background will be $\Omega_{\text{GW}}(f = 25 \text{ Hz}) = 1.1_{-0.7}^{+1.2} \times 10^{-9}$ [18]. The level is the same as the estimate from the initial observation [17], but the error has narrowed.

In order to estimate the stochastic background from binary black hole mergers one must take into account many factors.

For example, it is necessary to understand the mechanism by which these binaries are formed, which would then help to explain how often these sorts of mergers occur in the universe. The formation rate will depend on when this happens in the age of the universe, and the metallicity of the formation environment. The merger rate, as a function of redshift, will also be required.

A comprehensive explanation of how to calculate the contribution of binary black hole mergers to the stochastic background is given in [17], and presented here is a summary of that demonstration. Some set of intrinsic source parameters θ will describe the ensemble of binary black holes. These source parameters could be things like the masses and spins of the black holes. The distributions of these parameters are essentially unknown at present. However, the recent observations of binary black hole mergers by LIGO and Virgo [10, 11, 13, 170–172] and previous assumptions [173] allow for the division of this ensemble into different subsets. Consider a subset of binary black holes k described by parameters θ_k (for example, the mass and spin values). Call $R_m(z; \theta_k)$ the merger rate per comoving volume per unit source time; this depends on the formation rate of black hole binaries as a function of redshift and also the distribution of the time delays between binary black hole formation and merger [17, 174]. Then the total gravitational wave energy density spectrum for this particular class is (see, e.g. [81–83, 101, 175–178]):

$$\Omega_{\text{GW}}(f; \theta_k) = \frac{f}{\rho_c H_0} \int_0^{z_{\text{max}}} dz \frac{R_m(z, \theta_k) \frac{dE_{\text{GW}}}{df_s}(f_s, \theta_k)}{(1+z)E(\Omega_M, \Omega_\Lambda, z)}. \quad (17)$$

Note the term that accounts for cosmology, namely the dependence of how the comoving volume depends on redshift appears through $E(\Omega_M, \Omega_\Lambda, z) = \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}$. The spectral density of the energy of gravitational waves emitted at the source is $\frac{dE_{\text{GW}}}{df_s}(f_s, \theta_k)$. Then to calculate the total energy density a sum over all source classes k is performed [17].

The formation scenarios for binary black hole systems are important for predicting the expected rate of mergers over the history of the universe [179]. This would affect the predicted level of the subsequently produced stochastic background. In one scenario, the binary black holes are created as isolated binaries of massive stars in galactic fields [179–181]. An important observation for forming black holes similar to those observed in GW150914 is that there is a need for low metallicity, typically less than 10% of the solar metallicity; the initial stars would have masses in the range of 40–100 M_\odot [180]. The other formation channel for binary black holes is through dynamical interactions in dense stellar environments such as one might find in globular clusters [179, 182, 183]. Studies indicate that globular clusters can produce a significant population of massive black hole binaries that merge in the local universe, with most of the resulting binary black hole systems having total masses from 32 M_\odot to 64 M_\odot [182]. The formation rate as a function of redshift will ultimately affect the production of a stochastic background, and that will depend on how the binary black hole systems are formed. Clues as to the dominant formation channel may come through observation of the spins and

orbital eccentricities of a large number of gravitational wave events from binary black hole mergers [179].

Black holes that have been proposed to have been produced in the early universe are referred to as *primordial* black holes [184–189]. Primordial black holes have now also been suggested as the source of binary black hole systems in the universe. The possibility that dark matter could consist of primordial black holes has been raised after Advanced LIGO’s and Advanced Virgo’s observation of gravitational waves from binary black hole mergers. The observed masses for the binary black hole systems have been relatively large. There are claims that the mass window of $20\text{--}100 M_{\odot}$ cannot be excluded as the source of dark matter and could be the source of the LIGO–Virgo observations [190]. Given the presumed existence of primordial black holes, their implications for contributing to a binary black hole produced stochastic background has been investigated. One conclusion has been that the magnitude of the energy density from primordial black holes is much lower than that arising from the stellar-produced binary black hole mergers [191]. Other work has suggested that primordial black hole formation could be responsible for supermassive binary black hole mergers creating a stochastic background at the limit of what could be detected by pulsar timing experiments today [192].

Predictions suggest that there will be a binary black hole merger once every few tens of minutes in the observable universe [18]. The binary black hole merger signals will only appear within the LIGO–Virgo observation band for a period of the order of a second. As such, the binary black hole mergers form a non-Gaussian background of *popcorn* noise [193]. Through a mock data challenge, it has been verified that the standard stochastic search pipeline used by LIGO–Virgo is capable of efficiently detecting such a background [193], even if there are likely more efficient ways to do so. This is an ongoing field of research.

A stochastic background produced by binary black hole mergers will mask a cosmologically produced background. While an astrophysically produced stochastic background would provide a wealth of information, the observation of gravitational waves from the Big Bang is the Holy Grail of gravitational wave astronomy. For second generation gravitational wave detectors, such as Advanced LIGO, Advanced Virgo and KAGRA, it will be impossible to directly detect the majority of the binary black hole mergers over the history of the universe. However, the proposed third generation detectors, such as the Einstein Telescope [194] or the Cosmic Explorer [195], should be able to directly observe almost every stellar mass binary black hole merger in the observable universe. And whereas Advanced LIGO and Advanced Virgo should be observing a binary black hole stochastic background at the $\Omega_{\text{GW}} \sim 10^{-9}$ level, by removing this binary black hole foreground the third generation detection detectors could be sensitive to a cosmologically produced background at the $\Omega_{\text{GW}} \sim 10^{-13}$ level with 5 yr of observations [196]. With this sensitivity the third generation detectors will get into the realm where important cosmological observations could potentially be made [197].

Given the large number of signals present, it is interesting to consider the required data analysis challenges that will be faced by the third generation gravitational wave detectors [194, 195]. It is certainly probable that there will be overlapping signals. The Advanced LIGO–Advanced Virgo study describing the implications for a stochastic background given the observations of gravitational waves from binary black hole and binary neutron star mergers directly addresses the possibility of overlapping signals [18]. Given the expectations for the stochastic background produced by these compact objects a simulated time series of the signals was produced. Because binary neutron star gravitational wave signals occupy the observation band for a long time, these type of signals (from sources throughout the observable universe) overlap, whereas the binary black hole produced gravitational wave signals are in the observation band for shorter periods, and form a *popcorn* type of signal [18]. The predicted time between binary neutron star mergers in the observable universe is 13_{-9}^{+49} s, and assuming frequencies above 10 Hz, the number of overlapping signals at a given time is expected to be 15_{-12}^{+30} . For binary black hole mergers the predicted time between these events in the observable universe is 223_{-115}^{+352} s, while the number of overlapping signals at a given time is predicted to be $0.06_{-0.04}^{+0.06}$ [18].

The third generation gravitational wave detectors will have a lower frequency cutoff, probably 5 Hz. This means that the probability of signal overlap will be higher than for Advanced LIGO–Advanced Virgo since the signal will spend even more time in the detector. This is similar to the situation faced by LISA [32], which will need to deal with a very large number of overlapping signals (since, at these low frequencies, the source behavior is more like a continuous signal than a transient one); the same is true for other space based detectors [99, 100, 198]. Many methods have been developed to detect and characterize numerous overlapping gravitational wave signals with these space based gravitational wave detectors [199–203]; these types of methods to identify and then remove the compact binary merger gravitational wave signals will help the get the third generation gravitational wave detectors (and the space based detectors too) closer to measuring a cosmologically produced stochastic background [196].

2.6. Binary neutron stars

A stochastic background produced by binary neutron stars will definitely exist at some level. The dramatic observation by Advanced LIGO and Advanced Virgo of the binary neutron star inspiral GW170817 [14] has led to numerous important astrophysical observations. The associated short gamma ray burst, GRB 170817a, implies that binary neutron star mergers are the source of short gamma ray bursts, in general [46, 47]. The observation of the kilonova following the merger seems to confirm many predictions, including how the heaviest elements are created in the universe [15]. From the gravitational wave signal one can infer the luminosity distance to the source; then using the measured redshift of the host galaxy, a measurement of the Hubble constant could be made, independent of the cosmic distance ladder [204].

The observation of this binary neutron star merger also has important implications for the production of a stochastic gravitational wave background, and the ability of Advanced LIGO and Advanced Virgo to observe it [18]. This background would come from every binary neutron star merger throughout the observable universe; most of these are too small to be observed directly by LIGO and Virgo, but the background that they create may be detected. Using the observation of GW170817 and the total observing time by Advanced LIGO, the prediction for the energy density of a binary neutron star produced stochastic background will be $\Omega_{\text{GW}}(f = 25 \text{ Hz}) = 0.7_{-0.6}^{+1.5} \times 10^{-9}$. This can be compared with the predicted level of the binary black hole produced stochastic background of $\Omega_{\text{GW}}(f = 25 \text{ Hz}) = 1.1_{-0.7}^{+1.2} \times 10^{-9}$. The combination of the two gives the total astrophysically produced stochastic background, as predicted by the LIGO and Virgo observations, of $\Omega_{\text{GW}}(f = 25 \text{ Hz}) = 1.8_{-1.3}^{+2.7} \times 10^{-9}$ [18].

Then assuming the expected evolution of the sensitivity for Advanced LIGO and Advanced Virgo (as the detectors approach their design sensitivities) [205], it is estimated that the LIGO–Virgo network could observe this background with a signal-to-noise ratio of 3 after a total of approximately 40 months of observation in the Advanced LIGO–Advanced Virgo era (with observations starting with the first observing run, O1) [18]. Considering the uncertainties in the estimation of the background, and then taking the most optimistic assumptions, the astrophysical background might be observed at the 3σ level after 18 months of Advanced LIGO–Advanced Virgo era observations; this could then come during O3, the third observing run, scheduled to begin in the fall of 2018 [18]. The eventual detection of the astrophysically produced stochastic background by the LIGO–Virgo network is considered to be likely.

It is interesting to consider the nature of these two types of stochastic signals. When Advanced LIGO and Advanced Virgo reach their design sensitivities the low-frequency cutoff for observations will be 10 Hz. For the binary black hole produced stochastic background, the events come individually, once every 223_{-115}^{+352} s. The average duration of a signal in the interferometers’ observation band is approximately 14 s. The probability of two signals overlapping is therefore quite small. The average number of overlapping binary black hole gravitational wave signals is $0.06_{-0.04}^{+0.06}$. The situation is quite different for the binary neutron star produced stochastic background. For these signals the average length of time that they are in the observing frequency band is 190 s. These events arrive every 13_{-9}^{+49} s. Consequently, the average number of overlapping binary neutron star gravitational wave signals is 15_{-12}^{+30} [18]. A continuous background is created by the binary neutron star inspirals. But whether created by binary black holes or binary neutron stars, this astrophysically produced stochastic background is likely to be detected by the LIGO–Virgo network in the coming years.

2.7 Close compact binary stars

While systems like binary black holes and binary neutron stars are the sources of interesting gravitational wave signals, other

binary star systems will also produce gravitational waves. Close compact binary stars, most of which are white dwarf binaries, will produce thousands of signals that will be resolvable by LISA in the frequency band around a few 10^{-4} Hz to a few 10^{-2} Hz. In addition to binaries containing white dwarfs, there will be neutron stars and stellar-origin black holes in different combinations [32]. A background of gravitational waves will be formed by all of the unresolvable galactic [206] and extragalactic [207] binaries; the sum of all of the gravitational waves that are not individually resolvable will form a stochastic background which could make the observation of a cosmologically produced stochastic background challenging. It has long been recognized that LISA could directly observe gravitational waves from thousands of galactic binaries, while also having to contend with a stochastic background from unresolvable galactic and extragalactic binaries [208–210].

Having a mass model for the Milky Way helps to predict the distribution of close compact binary stars [211, 212]. This can then be used to predict the gravitational waves from these binary systems, including their distribution in the sky for LISA observations [213]. Knowing the distribution of galactic gravitational wave sources in the sky could help LISA to remove this signal and get to a cosmologically produced stochastic background, similar to what is done with observations of the CMB, namely the effort to remove the contamination by the galaxy or other foreground sources [214, 215]. LISA will certainly be able to produce a sky map of the galactic binaries producing gravitational waves in its observational band [216]. Further knowledge about galactic binary systems, including white dwarf binaries, will increase rapidly with the observations by Gaia and its creation of a three-dimensional map of the Milky Way [217–219].

The distribution of sources for gravitational waves from close compact binary stars can be seen in figure 5, along with the predicted sensitivity of LISA [32]. There will be thousands of galactic binaries in the LISA observation band that will be individually observable via gravitational wave emission. The points in the figure above the LISA sensitivity curve reflect predictions for individual observations with marked signal-to-noise ratio (SNR) > 7 . However there will be countless other binaries both in our galaxy and extragalactic that will contribute to an unresolvable gravitational background; this is also displayed in figure 5. It is predicted that in the LISA band, from 0.1 to 10 mHz, the gravitational wave background energy density from extragalactic binaries will be in the range $1 \times 10^{-12} < \Omega_{\text{GW}}(1 \text{ mHz}) < 6 \times 10^{-12}$ [207].

Whether it is the gravitational wave signals from thousands of directly observable galactic binaries, or the unresolved gravitational wave background from galactic and extragalactic binaries, these gravitational wave signals will create a tremendous data analysis challenge for the attempt by LISA to observe a cosmologically produced stochastic background. Research progress has shown that the thousands of individually detectable gravitational wave signals from galactic binaries can be removed from the search for a cosmologically produced stochastic background [199–201, 220–223]. The unresolvable gravitational waves from close compact binary stars need to be removed in the search for

the cosmologically produced stochastic background. Much progress has been made in addressing this problem [201, 223, 224], but further confirmation will need to be obtained in the coming years through LISA mock data challenges [225, 226].

While the numerous gravitational wave signals from close compact binary stars will present a data analysis challenge, some of these binary systems will be especially valuable for the LISA mission. Many of these systems have already been observed and studied electromagnetically; for LISA these are referred to as the *Verification Binaries*. These binary systems will produce gravitational wave signals that will be observed and used to confirm the calibration and sensitivity of LISA [227]. A comparison between the predicted and observed gravitational wave signals should provide significant confidence in the LISA observations and results. The verification binaries will also be used to test general relativity, including placing limits on the mass of the graviton [228].

LISA will also gain important information on binary systems in our galaxy through the observation of gravitational waves from ultra-compact binaries in the galaxy [229, 230]. These are binary systems consisting of two stars with an orbital period less than an hour. Of the order of 60 ultra-compact binaries have been identified via electromagnetic observations and these are thought to comprise white dwarfs, neutron stars, and stellar mass black holes; the double white dwarf binary J0651 has already been observed to have an orbital decay that is consistent with general relativity and loss of energy via the emission of gravitational waves [230]. The observation of these systems with gravitational waves will provide further tests of general relativity, and will also give information which will be helpful in explaining the formation and evolution of stellar binary systems [230].

Observations of gravitational waves from close compact binary stars are interesting in their own right. They provide a gravitational wave foreground and background containing much important astrophysical information. Ultimately if the close compact binary stars can be addressed by LISA (such as by subtracting signals from the data [202, 203], or accounting for them using Bayesian parameter estimation methods [199]) a sensitivity of $\Omega_{\text{GW}} \sim 10^{-12}$ could be achieved in the search for a cosmologically produced stochastic background of gravitational waves.

2.8. Supernovae

Common and powerful astrophysical events throughout the history of the universe will contribute to the stochastic background. If a supernova has some asymmetry, then gravitational waves will be produced. The emission of gravitational waves from supernovae has been studied in many ways. Numerical simulations provide some of the most comprehensive studies, but they are difficult and time consuming [34, 231–234].

There have been numerous studies which tried to address the level of a stochastic background produced by supernovae

in the universe. Population III stars¹ were formed in the early universe and had very large masses. Stars with high metallicity are more susceptible to mass loss via stellar winds [237]. Population III stars had very low metallicity (essentially zero) and, as such, were able to live their stellar lives with minimal mass loss. Population II stars had low metallicity compared with present day Population I stars. In [238] the authors consider Population III stars in the mass range of 100–500 M_{\odot} and Population II stars in the mass range of 8–40 M_{\odot} . Using redshift dependent formation rates for these stars, the expected evolution of these stars once created, and then the stars' death through supernovae, the resulting stochastic background is predicted. Assumptions are made as to the amount of energy released in gravitational waves in these supernovae. This study predicts a stochastic background that peaks in the LIGO–Virgo band, with $10^{-12} \leq \Omega_{\text{GW}} h^2 \leq 7 \times 10^{-10}$ in the 387–850 Hz frequency band. This stochastic background is dominated by gravitational waves from the supernovae of Population II stars [238].

Another study considers a stochastic background produced by the ringdown of black holes created via stellar core collapse [239]. Certainly this is only one of the different mechanisms for gravitational wave production in core collapse supernovae. Various models (including different star formation rates) predict a stochastic background of $10^{-10} \leq \Omega_{\text{GW}} \leq 5 \times 10^{-9}$ in the 50–1000 Hz frequency band. It is interesting to note that most of the gravitational wave production for this background comes from regions having redshifts of 1–2. This post-supernova black hole ringdown stochastic background is at a level that could be observed by the Advanced LIGO–Advanced Virgo network, or third generation detectors [239]. This level assumes that 10^{-6} – 10^{-4} of the rest mass of the black hole is converted into gravitational waves [239]. This efficiency assumption is probably quite optimistic.

Some of the members of the group who conducted the previous study extended their supernovae models to consider more general gravitational wave emission mechanisms [240]. The full supernova process and associated gravitational wave emission are very difficult to calculate. In the new study two models are considered. One considers the form of the gravitational wave signals produced by two- and three-dimensional supernova simulations. The form and frequency dependence of the gravitational wave emission from the core collapse supernova can be approximated [84, 241]. This can then be combined with predictions for star formation and eventual supernovae over the history of the universe. This then provides a prediction for a core collapse supernova produced stochastic background. Based on reasonable assumptions for the parameters in this model the stochastic background is predicted to be possibly as large as $\Omega_{\text{GW}} \sim 10^{-9}$ around 300 Hz, while

¹ Population I are young and metal-rich stars and are often found in the arms or spiral galaxies, such as in the Milky Way. Population II stars are very old, metal-poor and tend to be found in the center of galaxies or in galactic halos [235]. The hypothesized Population III stars would have essentially no metals, only the material present after the Big Bang (hydrogen, helium, and trace amounts of lithium and beryllium). Population III stars would be the oldest population of stars [236].

other parameter choices could reduce it to the $\Omega_{\text{GW}} \sim 10^{-12}$ level. The other model considered in this study concentrates on the low-frequency structure seen in the predicted gravitational wave emission from core-collapse supernova. This has been observed in the simulations from a number of groups, some of which suspect that it pertains to prompt convection. The most optimistic prediction for the stochastic background level for this model is $\Omega_{\text{GW}} \sim 10^{-10}$ in the 30–100 Hz band [240], and might be observable with third generation gravitational wave detectors [194, 195].

Since the gravitational wave production from supernovae is difficult to predict, the absence of a detection of a stochastic background can be used to constrain the average amount of gravitational wave emission from supernovae. Using the upper limits reported by initial LIGO and initial Virgo for the analysis of the scientific run S5 data it is possible to say that a supernova can only produce up to an upper limit in the range of $0.49\text{--}1.98 M_{\odot}c^2$ of energy in gravitational waves [85]. Future results on the upper limit of the gravitational wave energy density will provide further constraints. Note that since this study [85] was conducted the constraints imposed by LIGO on the stochastic background have improved by a factor of ~ 50 [51]. The constraints on the energy emitted in gravitational waves from supernovae will improve accordingly.

2.9. Pulsars and magnetars

Non-axisymmetric spinning neutron stars are expected to be a detectable source of gravitational waves [242, 243]. The radio observations from pulsars indicate that neutron stars rotate with periods that can be as rapid as milliseconds. Gravitational waves would be emitted if the neutron star is not perfectly spherical, namely if there is an asymmetry in its shape; such a deformation might be created by having toroidal magnetic fields within the neutron star [243, 244]. Another path for the production of gravitational waves would be the presence of a slight mountain on the neutron star surface. Such an effect could happen due to cracking of the crust through thermal effects [243, 245]. With such asymmetries or defects gravitational waves would be emitted at twice the rotation frequency of the neutron star.

The excitation of internal mechanical oscillation modes is another way for the symmetry of the neutron star to be broken, and for gravitational waves to be produced. There can be an interplay between the viscosity of the material within the neutron star and the emission of gravitational waves [246, 247]. It is also speculated that quadrupole mass currents can emit gravitational waves in such a way that the process actually amplifies the currents, leading to an unstable runaway process; these are associated with the so-called r-modes [243, 248–251].

Pulsars are numerous in our galaxy, and presumably in the universe. It was soon recognized that a stochastic background could be created by the sum of all neutron star produced gravitational waves in the universe. For example, one study [252] considers newly created neutron stars that are spinning rapidly. The neutron star loses energy and spins down via gravitational

wave emission. The r-mode instability [248–251] is responsible for the gravitational wave emission. The prediction from this study is an energy density of the stochastic background of $\Omega_{\text{GW}}h^2 \sim (2.2\text{--}3.3) \times 10^{-8}$ in the 500–1700 Hz frequency band. The results of this study are dependent on assumptions of the star formation rate, with the assumption that this peaks at a redshift of about $z \sim 1.3$ [252]. This study and results are similar (especially with respect to r-mode production of gravitational waves) to another [251], with results that are slightly different due to different assumptions about the star formation rate and its redshift dependence [251]. The star formation rate in the study of Owen *et al* [251] extends over the range $0 < z < 4$. The resulting predicted stochastic background is $\Omega_{\text{GW}}h^2 \sim 1.5 \times 10^{-8}$ at ~ 300 Hz, and diminishes for higher frequencies [251]. This corresponds to maximum gravitational wave production at a redshift of $z \sim 4$ [252].

A recent study has continued this avenue of research and investigated the stochastic background created by newly formed magnetars [88, 253]. A magnetar is a neutron star with an extraordinarily large magnetic field ($\sim 10^{14}\text{--}10^{15}$ G) [254]. Various equations of state for the neutron star matter are assumed, in addition to the merger rate for binary neutron star systems. Very strong magnetic fields for the newly formed magnetars are also assumed (from 10^{15} G to even 10^{17} G). The most optimistic results produced predictions of $\Omega_{\text{GW}} \sim 10^{-10}$ at ~ 100 Hz, $\Omega_{\text{GW}} \sim 10^{-9}$ at ~ 300 Hz, and $\Omega_{\text{GW}} \sim 10^{-8}$ at ~ 1000 Hz [88, 253].

It is also possible to calculate the gravitational wave production from all types of neutron stars, such as pulsars (typical magnetic field strengths, $\sim 10^8$ T), magnetars (very large magnetic fields, $\sim 10^{10}$ T, potentially creating ellipticities that enhance gravitational wave production), and gravitars (low magnetic field strengths, $< 10^8$ T, thereby making gravitational wave emission the dominant source of rotational energy loss) [86]. Different assumptions are made on the distribution of spins for the neutron stars. If the assumption is (admittedly optimistic) that all rotating neutron stars are gravitars, then the predicted gravitational wave emission is quite large, reaching $\Omega_{\text{GW}} \sim 10^{-7}$ at 1 kHz, or $\Omega_{\text{GW}} \sim 10^{-8}$ at 100 Hz. If on the other hand, the assumption is that neutron stars are essentially pulsars then the estimated stochastic background level is more pessimistic, with $\Omega_{\text{GW}} \sim 10^{-10}$ at 1 kHz, or $\Omega_{\text{GW}} \sim 10^{-13}$ at 100 Hz. For magnetars, and assuming their distribution is as described in [255], the prediction is that the resulting stochastic background would be $\Omega_{\text{GW}} \sim 10^{-8}$ at 1 kHz and $\Omega_{\text{GW}} \sim 10^{-10}$ at 100 Hz. The conclusion is that for realistic assumptions it will be difficult to detect this stochastic background, although with third generation detectors [194, 195] it might be possible [86].

The large number of neutron stars in the Milky Way, plus the fact that these neutron stars are relatively close, provides a means to constrain the average neutron star ellipticity based on the limits set on the stochastic background [256]. It is assumed that there are $10^8\text{--}10^9$ neutron stars in our galaxy [257]. Of these, it is predicted that of the order of $\sim 5 \times 10^4$ have rotation periods less than 200 ms, in which case they could produce gravitational waves in the observable band of

LIGO and Virgo, $f > 10$ Hz. The Advanced LIGO–Advanced Virgo network should be able to constrain the 1-sigma sensitivity to neutron star ellipticity to be $\sim 2 \times 10^{-7}$, which is also the limit derived from the two co-located initial LIGO detectors [55]. Third generation gravitational wave detectors [194, 195] may be able to constrain ellipticities to $\sim 6 \times 10^{-10}$ [256]. Theoretical studies predict that the largest possible ellipticity for a neutron star is $\sim 10^{-5}$ [258, 259].

The recent observation of the binary neutron star inspiral gravitational wave signal GW170817 [14] generated much interest as to the post-merger remnant. The total mass of the system was $2.74_{-0.01}^{+0.04} M_{\odot}$. The merger of the two neutron stars could have formed a black hole directly, in which case the black hole ringdown gravitational wave signal would be above 6 kHz. Another possibility is that a hypermassive neutron star could be formed, and it would survive for timescales of up to thousands of seconds before collapsing into a black hole. This hypermassive neutron star would survive through thermal gradients and differential rotation [260]. Another possibility is that a stable hypermassive neutron star is formed. In the short time after the merger the remnant will likely be excited, and emit gravitational waves in the 1 kHz–4 kHz regime [261–263]. LIGO and Virgo conducted a search for a post-merger gravitational wave signal associated with GW170817 [264]. A recent study considers a stochastic background created by such a post-merger remnant [265]. This study also discusses how future gravitational wave detectors could be designed and constructed at higher frequencies (1–4 kHz) to search for post-merger remnant signals, either for direct observation of an individual event or a stochastic background from these types of sources. The study claims that the combination of the binary neutron star inspiral signals and the post merger ringdown signals will contribute to a stochastic background of level $\Omega_{\text{GW}} \sim 10^{-9}$ from 1 to 3 kHz [265].

3. Summary of methods to observe or constrain a stochastic gravitational wave background

The search for a stochastic gravitational wave background is arguably one of the most important projects in cosmology and astrophysics. In contrast to the electromagnetic spectrum, gravitational waves will potentially provide a window to the earliest moments in the universe. In this section we review the methods by which one can attempt to observe the stochastic background. An extremely comprehensive review of the observational methods used and proposed to detect gravitational waves is given by Romano and Cornish [24].

3.1. LIGO–Virgo

The ground based gravitational wave detectors, LIGO and Virgo, have been attempting to measure the stochastic gravitational wave background since 2004 [51, 55, 90–95]. The magnitude of gravitational waves associated with the stochastic background will be random, so it will appear like noise in an individual detector. However, it will be coherent in two detectors (completely coherent for two co-located detectors,

with the coherence falling off with distance if the detectors are displaced from one another). The way to extract the stochastic background signal from two detectors is essentially outlined in equation (10). The correlation between the data from two gravitational wave detectors is more complicated due to their physical separation and misalignment. While this makes the calculation somewhat more involved, it is nonetheless straightforward to account for the presence of the stochastic background in both detectors [50].

The LIGO–Virgo data analysis method follows the prescription of Allen and Romano [89]. Instead of working in the time domain, as is the case with equations (9) and (10), one works in the frequency domain, using the Fourier transform of the signals, $\tilde{s}_1(f)$ and $\tilde{s}_2(f)$. An optimal filter is used to maximize the signal-to-noise ratio, but in order to do this, assumptions must be made on the frequency dependence of the signal. The search is described in terms of the energy density of the stochastic gravitational wave with respect to the closure density of the universe, as described by equations (8) and (14). Next, the frequency dependence of the energy density of the stochastic background is assumed to have the form

$$\Omega_{\text{GW}}(f) = \Omega_{\alpha} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}, \quad (18)$$

where f_{ref} is an arbitrary reference frequency. The search uses an estimator [51, 89]

$$\hat{Y}_{\alpha} = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \delta_T(f - f') \tilde{s}_1^*(f) \tilde{s}_2(f') \tilde{Q}_{\alpha}(f') \quad (19)$$

and variance

$$\sigma_{\hat{Y}}^2 \approx \frac{T}{2} \int_0^{\infty} df P_1(f) P_2(f) |\tilde{Q}_{\alpha}(f)|^2, \quad (20)$$

where $\delta_T(f - f')$ is a finite-time Dirac delta function, T is the observation time, $P_{1,2}$ are the one-sided power spectral densities for the detectors, and $\tilde{Q}_{\alpha}(f)$ is a filter function to optimize the search²,

$$\tilde{Q}_{\alpha}(f) = \lambda_{\alpha} \frac{\gamma(f) H_0^2}{f^3 P_1(f) P_2(f)} \left(\frac{f}{f_{\text{ref}}} \right)^{\alpha}. \quad (21)$$

The $\gamma(f)$ term is what is known as the overlap reduction function [50, 266]; this accounts for the reduction in sensitivity due to separation and relative misalignment between the two detectors used in the stochastic search. $\gamma(f) = 1$ if the detectors are co-located and co-aligned, and diminishes otherwise. Note that it is actually the magnitude, $|\gamma(f)|$, that is the most important; a rotation of a detector by 90° will not affect the sensitivity of the search for the stochastic background.

3.2. Results from Advanced LIGO observing run O1

Advanced LIGO's first observing run went from September 2015 to January 2016. The data from the two Advanced LIGO

² The Hubble constant appears explicitly, rather than being absorbed into λ_{α} , to emphasize that the estimator for Ω_{GW} depends on the measured value of H_0 .

detectors, LIGO Hanford and LIGO Livingston, were used in the search for a stochastic background. Data quality cuts removed problematic times and frequencies from the analysis. In total, 29.85 d of coincident data were analyzed. No stochastic background was detected. The dramatic improvement in the upper limit on the stochastic background energy density was important, but not the most important stochastic background outcome of observing run O1. The observation of the gravitational waves from stellar mass binary black hole mergers [4, 8, 9] implies that these events are far more numerous in the universe than previously expected. In fact, it is likely that the stochastic background produced from these type of events will be at the level of $\Omega_{\text{GW}} \sim 10^{-9}$ in the observation band of Advanced LIGO and Advanced Virgo [17]. See figure 2.

3.2.1. O1 isotropic results. Assuming that the frequency dependence of the energy density of the stochastic background is flat, namely $\alpha = 0$, the constraint on the energy density is $\Omega(f) < 1.7 \times 10^{-7}$ with 95% confidence within the 20 Hz–86 Hz frequency band [51]. This is a factor of 33 better than the upper limit set by initial LIGO and initial Virgo [95]. Assuming a spectral index of $\alpha = 2/3$ the constraint on the energy density is $\Omega(f) < 1.3 \times 10^{-7}$ with 95% confidence within the 20–98 Hz frequency band, while for $\alpha = 3$ it is $\Omega(f) < 1.7 \times 10^{-8}$ in the 20–300 Hz band [51] (the reference frequency is $f_{\text{ref}} = 25$ Hz when $\alpha \neq 0$). Figure 3 provides the O1 stochastic background results, as well as constraints from from previous analyses, theoretical predictions, the expected sensitivity at design sensitivity for Advanced LIGO and Advanced Virgo, and the projected sensitivity of the proposed Laser Interferometer Space Antenna (LISA) [32]. The O1 results will be used to limit cosmic string parameters, similar to what was done with initial LIGO and initial Virgo [38, 94].

3.2.2. O1 anisotropic results. Within the LIGO–Virgo observational band it is expected that the stochastic background will be essentially isotropic. However, LIGO and Virgo have decided to look for a stochastic background that would be anisotropic. Such an anisotropic background could provide even more information about the early universe, or the astrophysical environment in our region of the universe. Using the recent O1 data there have been three different types of searches for an anisotropic background [54]. To look for extended sources, LIGO and Virgo use what is known as the spherical harmonic decomposition [267]. In order to search for point sources, a broadband radiometer analysis is used [268, 269]. Finally, LIGO and Virgo employed a narrowband radiometer search to look for gravitational waves in the direction of interesting objects in the sky, such as the galactic center, Scorpius X-1 and SN 1987A.

An anisotropic stochastic background was not observed with the Advanced LIGO O1 data, but important upper limits were set [54]. For broadband point sources, the gravitational wave energy flux per unit frequency was constrained to be $F_{\alpha,\Theta} < (0.1\text{--}56) \times 10^{-8}$ erg cm⁻² s⁻¹ Hz⁻¹($f/25$ Hz) ^{$\alpha-1$} depending on the sky location Θ and the spectral power index α . For extended sources, the upper limits on the fractional gravitational wave energy density required

to close the Universe are $\Omega(f, \Theta) < (0.39\text{--}7.6) \times 10^{-8}$ sr⁻¹($f/25$ Hz) ^{α} , again depending on Θ and α . The directed searches for narrowband gravitational waves from Scorpius X-1, Supernova 1987 A, and the Galactic Center had median frequency-dependent limits on strain amplitude of $h_0 < (6.7, 5.5, \text{ and } 7.0) \times 10^{-25}$ respectively, for the most sensitive detector frequencies 130–175 Hz. See [54] for further details.

3.2.3. Tests of general relativity with the stochastic gravitational-wave background. LIGO and Virgo have used the recent observation of gravitational waves from binary black hole and binary neutron star inspirals to test general relativity [9, 46, 270]. The LIGO–Virgo stochastic background search has also been extended in order to test general relativity. Assuming that general relativity is the correct description of gravitation, there is no reason to expect extra polarizations of gravitational waves, nor extra polarizations in the stochastic background; however, LIGO and Virgo have the ability to search for these modes, and will do so. With general relativity there are only two possible polarizations for gravitational waves, namely the two tensor modes. Alternative theories of gravity can also generate gravitational waves with scalar or vector polarizations [271]. The observation of the gravitational waves from the binary black hole merger by the three detectors of the Advanced LIGO–Advanced Virgo network, GW170814, allowed for the first direct test as to whether the polarization of gravitational waves obeys the predictions of general relativity; from this observation, the tensor-only polarizations of general relativity are preferred [13].

Since there are six possible polarization modes (see figure 4), Advanced LIGO (with only two detectors, that are essentially co-aligned with respect to each other) cannot identify the polarization of short duration gravitational wave signals [9, 24, 271], such as those that have been recently observed [4, 8, 9]. A minimum of six detectors would be necessary to resolve the polarization content (scalar, vector and tensor) of a short duration gravitational wave [271]. A search for long duration gravitational waves, such as those from rotating neutron stars or the stochastic background by the two Advanced LIGO detectors, can directly measure the polarizations of the gravitational waves [24, 272–275]. A detection of a stochastic background by Advanced LIGO and Advanced Virgo would allow for a verification of general relativity that is not possible with short duration gravitational wave signals.

The LIGO–Virgo search for a stochastic background has now been expanded to a search for six polarizations: two tensor modes, two vector modes, and two scalar modes [275, 276]. This has been applied to Advanced LIGO Observing Run O1 data [276]. In future observing runs, the addition of Advanced Virgo to the network will not improve detection prospects (because of its longer distance displacement from the LIGO detectors); however, it will improve the ability to estimate the parameters of a stochastic background of mixed polarizations. The eventual inclusion of KAGRA [277] and LIGO-India [278] will further expand the ability to resolve different polarizations of the stochastic background,

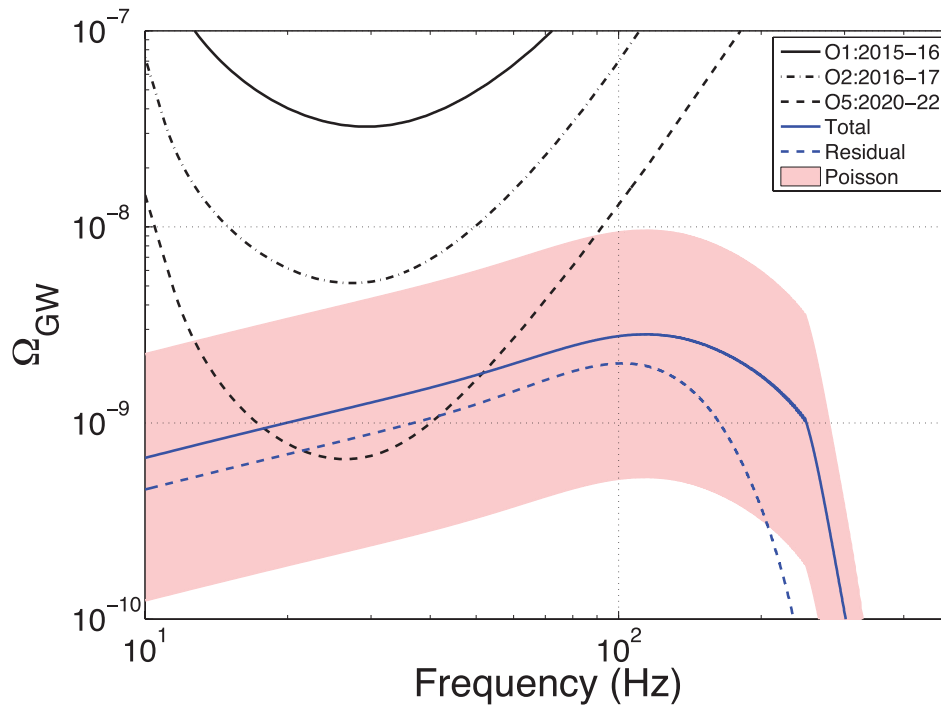


Figure 1. As presented in [17], the predicted Advanced LIGO–Advanced Virgo network sensitivity to a stochastic background produced by binary black holes that were formed through binary stellar evolution. Displayed are the energy density spectra (solid for the total background; dashed for the residual background, excluding resolved sources, assuming final Advanced LIGO and Advanced Virgo sensitivity). The pink region represents the uncertainty in the estimation. The black curves (O1, O2 and O5) display the 1σ sensitivity of the Advanced LIGO–Advanced Virgo network expected (at the time of the publication of [17]) for the observation runs O1 and O2, and the design sensitivity (2 yr of observation in O5). Reprinted figure with permission from [17], Copyright (2016) by the American Physical Society.

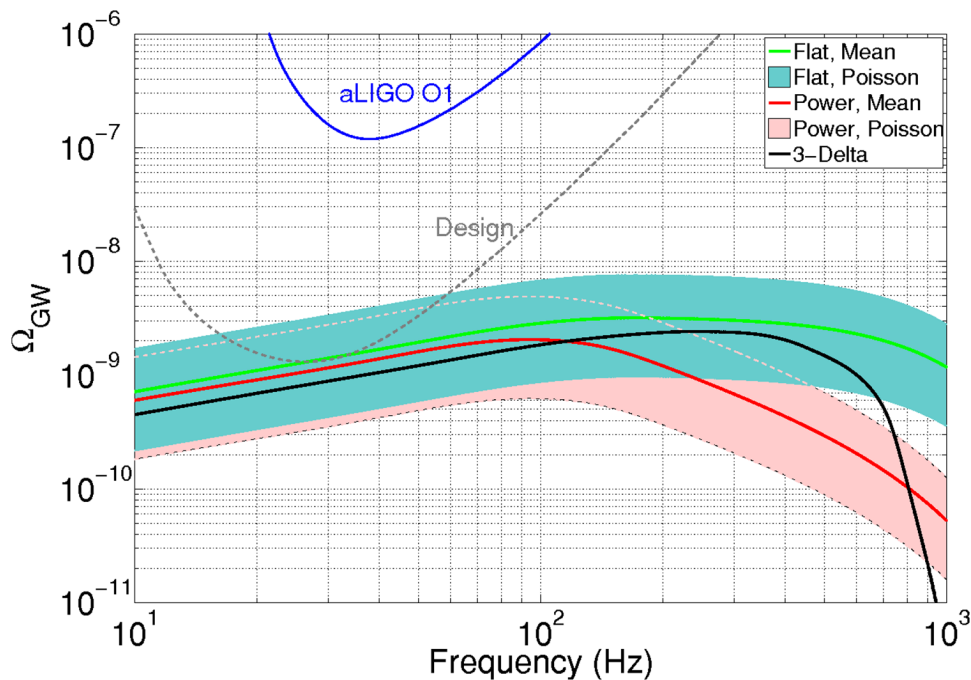


Figure 2. The range of potential spectra for a binary black hole background assuming the flat-log, power-law, and three-delta mass distribution models described in [9, 179], with the local rate derived from the O1 observations [9]. Also displayed is the O1 sensitivity and the projected ultimate design sensitivity for Advanced LIGO and Advanced Virgo. Reprinted figure with permission from [51], Copyright (2017) by the American Physical Society.

and further test general relativity. Bayesian parameter estimation techniques have been developed in order to search for tensor, vector and scalar polarizations in the LIGO–Virgo data [275].

For the Advanced LIGO O1 data, there has been a search for tensorial gravitational waves, vector gravitational waves, and scalar gravitational waves [276]. While no signal was detected, upper limits have been placed on the energy density of each of

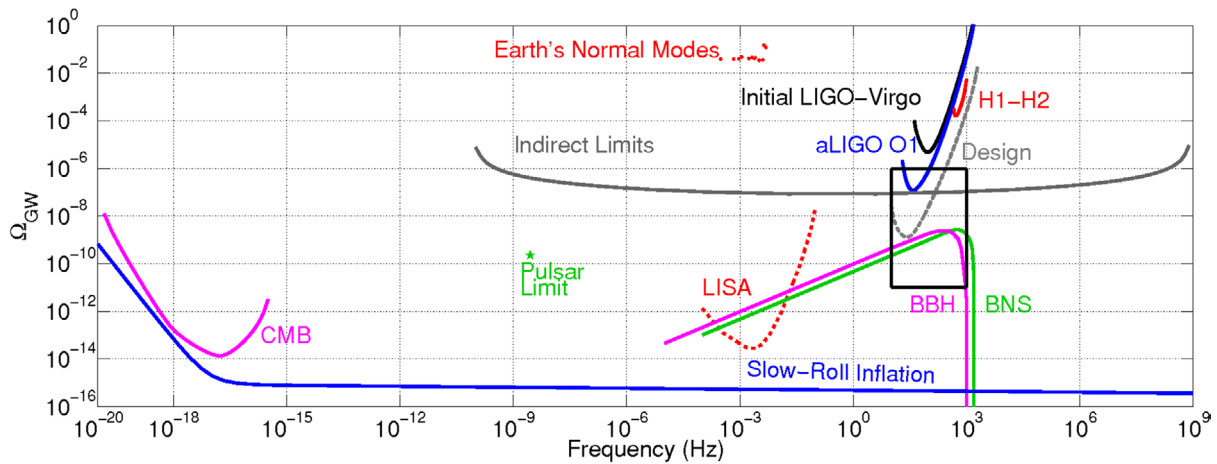


Figure 3. Constraints on the stochastic background, as well as various predictions, across over 29 decades in frequency. Displayed are the limits from the final science run of initial LIGO–Virgo, the co-located detectors at Hanford (H1–H2) during run S5, Advanced LIGO for O1, and the expected design sensitivity of the Advanced LIGO–Advanced Virgo detector network assuming two years of coincident data. Also shown are the constraints on the energy density of the stochastic background from other observations: CMB measurements [61], indirect limits from the cosmic microwave background (CMB) and Big-Bang nucleosynthesis [45, 96], pulsar timing [45], and from the Earth’s normal modes [97]. The predicted stochastic background from binary black holes (BBH) [17] and binary neutron stars (BNS) [193] are displayed. Also given is the predicted sensitivity for the proposed space-based detector LISA [32]. Displayed in figure 2 is the region in the black box in more detail. Finally, the stochastic gravitational-wave background predicted from slow-roll inflation is displayed; this result is consistent with the Planck results [44] and for this plan a tensor-to-scalar ratio of $r = 0.11$ is used. Reprinted figure with permission from [51], Copyright (2017) by the American Physical Society.

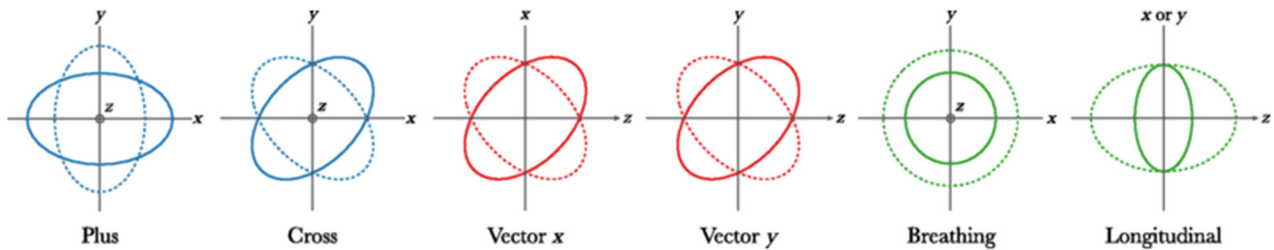


Figure 4. The effect of different possible polarizations of gravitational waves on a ring of freely falling test particles. The six gravitational-wave polarizations are allowed with general metric theories of gravitation. The gravitational waves are assumed to be propagating in the z direction (out of the page for the plus, cross, and breathing modes; to the right for the vector- x , vector- y , and longitudinal modes). While general relativity allows only for two tensor polarizations (plus and cross), other theories allow for two vector (x and y) and/or two scalar (breathing and longitudinal) polarizations. Reprinted figure with permission from [280], Copyright (2017) by the American Physical Society.

these stochastic backgrounds. This search assumed log-uniform priors³ for the energy density in each polarization; note that in the O1 Advanced LIGO results reported in [51] it was assumed that the prior on the energy density was uniform in a particular band. With 95% credibility, the limit for the energy density of the tensor modes is $\Omega_{\text{GW}}^{\text{T}} < 5.6 \times 10^{-8}$, for the vector modes $\Omega_{\text{GW}}^{\text{V}} < 6.4 \times 10^{-8}$, and scalar modes $\Omega_{\text{GW}}^{\text{S}} < 1.1 \times 10^{-7}$; for these limits the reference frequency is 25 Hz [276].

3.3. LIGO co-located detectors

In principle the best chance of detecting a stochastic background would be with two co-located and co-aligned detectors. In this case the overlap reduction function $\gamma(f)$ [50, 266], would be equal to 1 for all of the frequencies in the search. For the first five scientific runs of initial LIGO, S1–S5, there were two interferometers operating at the LIGO Hanford site. H1 was the 4 km interferometer, while H2 was the 2 km

interferometer. These two detectors were co-aligned and co-located, and operated within the same vacuum system. Using the LIGO H1 and H2 S5 data a search was conducted for a stochastic background [55].

In reality this search proved to be very difficult. Common noise was coherent in both detectors. As such, the correlation that was measured between the gravitational wave data from H1 and H2 was corrupted by the presence of coherent noise. This was especially true at low frequencies, $f < 460$ Hz. However, at higher frequencies it was possible to conduct the search. For the band of 460–1000 Hz, a 95% confidence-level upper limit on the gravitational-wave energy density was found to be $\Omega_{\text{GW}}(f) < 7.7 \times 10^{-4}(f/900 \text{ Hz})^3$ [55]. These continue to be the best upper limits in this frequency band [51].

3.4. Correlated magnetic noise in global networks of gravitational-wave detectors

A search for the stochastic background uses a cross-correlation between the data from two detectors. Inherent in such

³This is a uniform prior between $\log(\Omega_{\text{min}})$ and $\log(\Omega_{\text{max}})$. For the analysis of the Advanced LIGO O1 data $\Omega_{\text{min}} = 10^{-13}$ and $\Omega_{\text{max}} = 10^{-5}$ [276].

an analysis is the assumption that the noise in one detector is statistically independent from the noise in the other detector. Correlated noise would introduce an inherent bias in the analysis. It is for this reason that the data from two separated detectors is used. See section 3.3 for the discussion of co-located detector measurement [55].

The LIGO and Virgo detectors' sites are thousands of kilometers from one another, and the simple assumption is that the noise in the detectors at these sites is independent from one another. However, this assumption has been demonstrated to be false for magnetic noise. The Earth's surface and the ionosphere act like mirrors and form a spherical cavity for extremely low frequency electromagnetic waves. The Schumann resonances are a result of this spherical cavity, and resonances are observed at 8, 14, 20, 26,... Hz [279]. Most of these frequencies fall in the important stochastic background detection band (10 Hz–100 Hz) for Advanced LIGO and Advanced Virgo. The resonances are driven by the 100 or so lightning strikes per second around the world. The resonances result in magnetic fields of order 0.5–1.0 pT Hz^{-1/2} on the Earth's surface [279]. In the time domain, 10 pT bursts appear above a 1 pT background at a rate of ≈0.5 Hz [280].

This magnetic field noise correlation has been observed between magnetometers at the LIGO and Virgo sites [56]. Magnetic fields can couple into the gravitational wave detectors and create noise in the detectors' output strain channels. It has been determined that the correlated magnetic field noise did not affect the stochastic background upper limits measured by initial LIGO and Virgo, but it is possible that they could contaminate the future results of Advanced LIGO and Advanced Virgo [281]. If that is the case, then measures must be taken to try to monitor the magnetic fields and subtract their effects. This could be done, for example, via Wiener filtering [58, 281, 282]. Low noise magnetometers are now installed at the LIGO and Virgo sites in order to monitor this correlated magnetic noise. The data from these magnetometers will be used for Wiener filtering if it is necessary for the stochastic background searches. In addition to long term magnetic noise correlations, short duration magnetic transients, produced from lightning strikes around the world, are seen to be coincidentally visible at the detector sites and could affect the search for short duration gravitational wave events [57].

3.5. Future observing runs for LIGO and Virgo

Advanced LIGO has completed its first observing run, and the results of the search for a stochastic background have been published [51, 54]. At the time of writing Advanced LIGO has completed its second observing run, with Advanced Virgo joining it for the last month. Over the next few years further observing runs will happen as Advanced LIGO and Advanced Virgo approach their target sensitivities [283]. At their target sensitivities LIGO and Virgo should be able to constrain the energy density of the stochastic background to approximately $\Omega_{\text{GW}} \sim 1 \times 10^{-9}$ (in the 10 Hz–100 Hz band) with a year of coincident data, while 3 yr of data will give a

limit of $\Omega_{\text{GW}} \sim 6 \times 10^{-104}$. At this point it is likely that LIGO and Virgo could observe a stochastic background produced by binary black holes and binary neutron stars [17, 18, 51]. Various cosmological models [113, 115, 284, 285] or cosmic strings [286–289] might produce a detectable stochastic background at this level as well. Similar sensitivity advances will also be made with the directional searches as Advanced LIGO and Advanced Virgo reach their target sensitivities. In fact, the addition of Advanced Virgo to the network, with its long distance displacement from the LIGO sites, will make a further important contribution to the directional searches and their ability to map the sky [54]. One can expect to see many important results pertaining to the search for a stochastic background from LIGO and Virgo in the coming years.

3.6. Laser Interferometer Space Antenna—LISA

A way to avoid the many deleterious noise sources found on the Earth is to put a gravitational wave detector in space. This is the idea behind the laser interferometer space antenna (LISA) [31, 32]. The LISA mission has been accepted by the European Space Agency (ESA), with the National Aeronautics and Space Administration (NASA) participating as a junior partner. The current plan is for a 2034 launch, with a mission lasting 4 yr, with the possibility of an extension to 10 yr of total observation time.

LISA will consist of three satellites in an equilateral triangle configuration, separated from one another by 2.5×10^6 km. This will allow for three gravitational wave *interferometers*. Strictly speaking, these will not be interferometers of the kind used by LIGO and Virgo. Of the order of ~1 W of laser light will be emitted from one satellite, while only picowatts will be received by the other. As such, the phase of the incoming beam will be measured, and the re-emitted light will have its phase set accordingly [32]. At low frequencies only two of the interferometers' data streams will be independent [290].

LISA Pathfinder has demonstrated that much of the technology required for the LISA mission can meet the requirements for its success [291, 292]. For example, with LISA Pathfinder the relative acceleration noise of two test masses was measured to be $(1.74 \pm 0.05) \text{ fm s}^{-2} \text{ Hz}^{-1/2}$ above 2 mHz and $(6 \pm 1) \times 10 \text{ fm s}^{-2} \text{ Hz}^{-1/2}$ at 20 μHz. This level of relative acceleration noise meets the requirements for the LISA mission.

LISA will be able to observe gravitational waves from any direction in the sky. It will also be generally sensitive to both polarizations of gravitational waves from any direction. The operating band for LISA will extend from frequencies smaller than 10^{-4} Hz to those greater than 10^{-1} Hz. This will be an important observation band for observations, with many interesting signals predicted [32, 293].

One of the important signal sources for LISA will be the stochastic background. Certainly all of the compact galactic binaries will produce a stochastic background that will be

⁴ Note that the predicted evolution of the LIGO–Virgo sensitivity for the stochastic background search, from O1 to reaching design sensitivity, is displayed in figure 1 of [18].

significant for LISA; so significant, that it could mask other more interesting signals. Various methods have been suggested for accounting for galactic binary signals within the LISA data [199, 200, 220–222, 294].

Other important sources for a stochastic background will include binary black hole systems throughout the universe. The detections by LIGO and Virgo of gravitational waves from binary black hole inspirals implies that there will be a stochastic background from these systems from throughout the history of the universe [17]. This stochastic background will also be potentially observable by LISA. The energy density of this background will vary as $\Omega_{\text{GW}}(f) \propto f^{2/3}$. The predicted stochastic background was $\Omega_{\text{GW}}(f) = 1.1_{-0.9}^{+2.7} \times 10^{-9} (f/25 \text{ Hz})^{2/3}$. An assumption of the worst case scenario gives a background at the $\Omega_{\text{GW}} \sim 10^{-10} (f/25 \text{ Hz})^{2/3}$ level. If LISA observes a stochastic background it will be important for it to also be able to measure its spectral variation. A goal of the LISA mission is to make measurements of this stochastic background in two bands, $0.8 \text{ mHz} < f < 4 \text{ mHz}$ and, $4 \text{ mHz} < f < 20 \text{ mHz}$, each with a signal-to-noise ratio of 10 assuming 4 yr of integration time. This should be achievable by LISA [32].

Certainly a cosmologically produced stochastic background would be the most interesting as it would give direct evidence about the universe at its earliest moments. For example, a first-order phase transition in the energy range from hundreds of GeV to one TeV would produce gravitational waves that would fall within LISA's observation band [153, 154]. By measuring the spectral shape it will be possible to begin to decipher the source of the background. For example, LISA hopes to detect stochastic backgrounds produced by inflation [163], first-order phase transitions [154], and cosmic strings [295]. In order to have sufficient sensitivity to make statements about the spectral characteristics of the stochastic background, LISA is being designed so that its sensitivity is sufficient to achieve measurements of $\Omega_{\text{GW}} = 1.3 \times 10^{-11} (f/10^{-4} \text{ Hz})^{-1}$ for $0.1 \text{ mHz} < f < 20 \text{ mHz}$, and $\Omega_{\text{GW}} = 4.5 \times 10^{-12} (f/10^{-2} \text{ Hz})^3$ for $2 \text{ mHz} < f < 200 \text{ mHz}$. Again, this assumes 4 yr of observation [32]. Because there are three detectors, each sharing an arm and laser beam with its neighbor, there will be correlations in the signals and the noise. It will be helpful to this search that a *null-stream* can be created; namely an output channel where there is no signal (to some approximation). However, it is possible that correlated noise could affect the data; this would be especially problematic for a search for a stochastic background. LISA will take the data from the three interferometers and recombine them to create three different channels using *Time Delay Interferometry* [296], a way to minimize laser noise when the arm lengths for the interferometers are unequal. Nominally the noise and signals will then be uncorrelated. Between correlated noise and the galactic binaries it will be a challenge for LISA to achieve the $\Omega_{\text{GW}} \sim 10^{-12}$ level, but certainly not impossible either. Much research is already underway in order to achieve the LISA goals for measuring or setting limits on a stochastic background [32].

The sensitivity of the proposed LISA 3-detector system with $2.5 \times 10^6 \text{ km}$ arms is presented in figure 5 [32]. The

signal sources that are expected to be observed by LISA are also presented. It is important to note the presence of the close compact binaries, as described in section 2.7. Those binaries producing gravitational waves above the LISA sensitivity (marked with $\text{SNR} > 7$) will be individually resolvable, and in principle can be removed so that they do not contaminate the LISA stochastic background search [199–201, 220–223]. However, the sum of all other binaries will produce a gravitational wave background that must be addressed in a search for a cosmologically produced stochastic background [201].

3.7. DECI-hertz Interferometer Gravitational wave Observatory—DECIGO

Another proposed space based gravitational wave detector is the DECI-hertz Interferometer Gravitational wave Observatory, a Japanese project [99, 100]. Similar to LISA, DECIGO will consist of three satellites, in an equilateral triangle configuration, but with a distance separation of 1000 km. It will also be in a heliocentric orbit. The light traveling between each spacecraft will be within a Fabry–Pérot cavity, similar to what is done in the arms of LIGO and Virgo. The proposal is for four DECIGO clusters (with a DECIGO cluster consisting of three satellites in a 1000 km equilateral triangle configuration). Two of the DECIGO clusters will be overlapping, with the two equilateral triangles displaced from one another by a rotation of 30° . This close proximity should make DECIGO especially sensitive to a stochastic background.

DECIGO's operating frequency band will be 0.1 Hz to 10 Hz. This will form an important bridge in frequency space between LISA, and the LIGO–Virgo–KAGRA ground based network. This frequency band is particularly advantageous in that the contamination of the signals from white dwarf binaries will be extremely low, giving a window for a search for a cosmologically produced stochastic background [207]. Because of this reduced white dwarf binary foreground, and the sensitivity of DECIGO, it could be possible to achieve a detection limit for a stochastic background search of $\Omega_{\text{GW}} \sim 2 \times 10^{-16}$ with three years of observations. This impressive sensitivity could provide a direct observation of gravitational waves produced during inflation [20]. In addition, with DECIGO it could also be possible to measure the Stokes V parameter, namely a measure of the circular polarization [299]. A measured asymmetry in right-handed and left-handed polarizations of a stochastic background could indicate parity violation in the early universe. An adjustment in the positions of the DECIGO clusters will allow DECIGO to be sensitive to an asymmetry in the right-handed and left-handed gravitational waves, as quantified by the Stokes V parameter [299]. Initial LIGO data has been used to search for a parity violation, but with no detected stochastic background the results are consistent with $\Pi = 0$, with $\Pi = \pm 1$ representing fully right- or left-handed gravitational waves polarizations [300]. Since DECIGO could in principle measure a cosmologically produced stochastic background, it could then subsequently search for these signatures of parity violation.

The current planning for the mission estimates that DECIGO will be launched in the 2030s [100]. In preparation

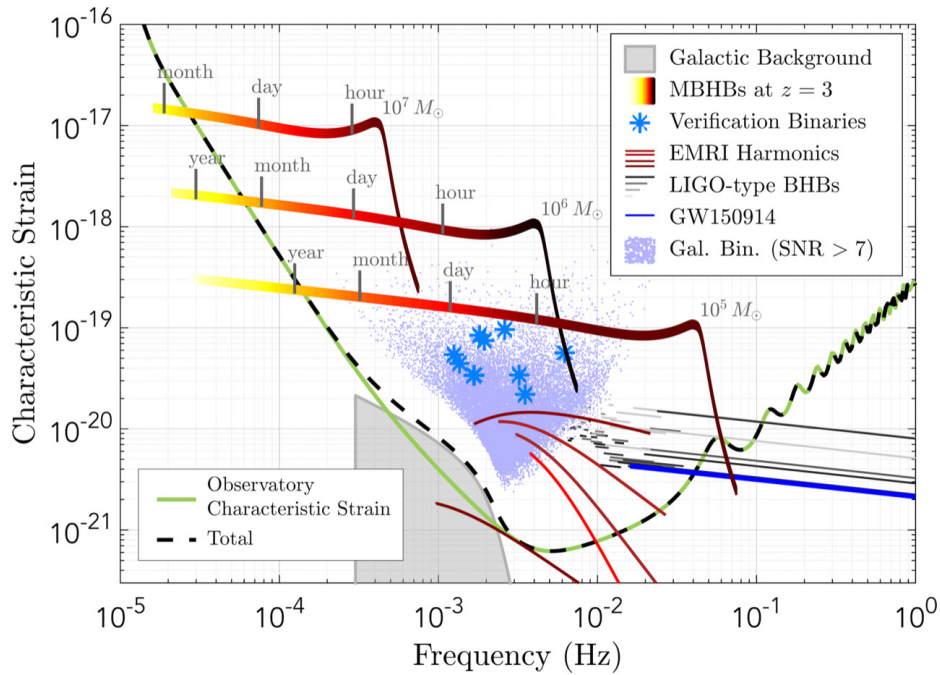


Figure 5. The sensitivity (in terms of characteristic strain [297, 298]) of the proposed LISA 3-detector system with 2.5×10^6 km arms [32]. Numerous sources that are expected to be observed by LISA are displayed. Especially important for the search for a stochastic background will be the galactic background (see section 2.7). Thousands of galactic binaries in LISA will produce signals with $\text{SNR} > 7$, and will be individually resolvable. Some of these systems are well known and have already been studied; these will be the so-called *Verification Binaries*, that will produce gravitational wave signals that will help to confirm the sensitivity and calibration of LISA. However, countless other binary systems will contribute to a gravitational wave background that will complicate the LISA search for a cosmologically produced stochastic background [201, 223]. This background is also displayed in this figure. Many other predicted signals for LISA are displayed, including massive black hole binaries (MBHBs, including GW150914), black hole binary systems that can be observed with Advanced LIGO and Advanced Virgo (LIGO-type BHBs), and extreme mass ratio inspirals (EMRIs). See [32] for more details on these signal sources. Reproduced from [32] with permission. <https://arxiv.org/abs/1702.00786>.

for this ambitious mission a smaller version of DECIGO is being planned for launch in the late 2020s, called B-DECIGO. This will consist of three satellites, but with a separation of 100 km, and orbiting the Earth. B-DECIGO is intended to demonstrate the technology needed for the full DECIGO mission, but it could detect gravitational waves in its own right [100].

3.7.1. Big Bang Observer and other space mission proposals. A project similar to DECIGO is the Big Bang Observer (BBO) [198]. Like DECIGO, it would have a triangular configuration, but with arm lengths of 5×10^4 km. With two overlapping triangular clusters a cross-correlation can be made between independent detector data sets [301]. BBO is designed to look for a cosmologically produced stochastic background, with a sensitivity of $\Omega_{\text{GW}} \sim 10^{-17}$ in the 0.03 Hz–3 Hz frequency band [198, 301]. This important frequency band should be free of astrophysical contamination [301].

Cornish and collaborators have explored various modifications to the LISA–DECIGO–BBO designs, especially the concept of two overlapping triangular clusters [302]. The cross-correlation of the data from the two overlapping (but independent) detectors creates an opportunity to achieve a sensitivity whereby gravitational waves from inflation could be detected. A major goal would be to search for a stochastic background around a μHz , thereby operating in a regime with

minimal contamination from astrophysical sources [302]. The proposal would be for a successor to LISA, namely a LISA II with arm lengths of $\sqrt{3}$ AU. LISA II is proposed to be a system of 6 spacecraft in a configuration of two equilateral triangles, essentially two overlapping LISA systems [302]. The $\sqrt{3}$ AU large arm lengths require an orbit farther out, which results in reduced thermal effects because of the diminished solar heating. In addition, a relative acceleration noise for the proof masses is assumed to be at the level of $\delta a \sim 3 \times 10^{-16} \text{ m s}^{-2}$. And while the recent observations of the relative acceleration of the proof masses for LISA Pathfinder were impressive [291, 292], an improvement will still be necessary, especially at this low frequency of a μHz . Given the assumptions for the detectors' performance it is speculated that the LISA II design could observe a stochastic background at the level of $\Omega_{\text{GW}} \sim 4 \times 10^{-13}$. Even more ambitious would be the LISA III design, with arm lengths of 35 AU. In this case the sensitivity to a stochastic background could reach $\Omega_{\text{GW}} \sim 2 \times 10^{-18}$ at 10^{-8} Hz. This would certainly be sufficient to observe gravitational waves from inflation [20].

3.8. Fermilab Holometer

The Fermilab Holometer consists of two Michelson interferometers that are nearly overlapping (a separation of 0.635 m), with arm lengths of 39.2 m [303]. The holometer was

constructed with the goal of attempting to observe correlations in space-time variations. It was speculated [304–306] that this could be a consequence of quantum gravity. However the two co-aligned and co-situated interferometers also provide a unique means to try to measure a stochastic background at MHz frequencies [307].

The holometer has recently demonstrated that its strain sensitivity $h(f) = \sqrt{S_h}$ (see equation (11)) is better than $10^{-21} \text{ Hz}^{-1/2}$ in the 1 MHz to 13 MHz band. With 130h of coincident data between the two interferometers a 3σ limit on the energy density of the stochastic background was made, $\Omega_{\text{GW}} < 5.6 \times 10^{12}$ at 1 MHz, and $\Omega_{\text{GW}} < 8.4 \times 10^{15}$ at 13 MHz [307]. These are the best limits to date in this high frequency band for a direct measurement, although Big Bang nucleosynthesis [308], CMB observations [128, 309] and indirect limits [45] do place much better constraints at these frequencies.

3.9. Pulsar timing

Pulsars are like clocks in space. These are rapidly rotating neutron stars with large magnetic fields. It is presumed that there is a misalignment between the magnetic field dipole axis and the rotation axis. As such, the sweeping magnetic field creates a regularly arriving radio pulse. These pulses were first detected on Earth in 1967 and reported in *Observation of a Rapidly Pulsating Radio Source* [310]. It was quickly deciphered that these radio signals were coming from rapidly rotating neutron stars, namely *pulsars* [311, 312].

Sazhin [313] and Detweiler [314] were the first to recognize that the regularity of the signals received from pulsars could be used to search for gravitational waves. For the detection of gravitational waves, one can consider a pulsar and an observer on Earth to be analogous to the two ends of a single interferometer arm. For long gravitational wave periods ($T \sim 1 \text{ yr}$) the energy density of the stochastic background can be expressed as

$$\Omega_{\text{GW}}(f) = \frac{2\pi^2}{3H_0^2} A_{\text{GW}}^2 f_{\text{yr}}^2 \left(\frac{f}{f_{\text{yr}}}\right)^{n_t}, \quad (22)$$

where A_{GW} is the characteristic strain amplitude at the reference frequency $f_{\text{yr}} = 1 \text{ yr}^{-1}$, and n_t is the spectral index; see equation (6) of [45]. In addition, see [24] for a comprehensive description of how one can extract a gravitational wave signal from the pulsar timing data.

Needless to say, while the signal from a pulsar can be regular, numerous effects can modify the phase of the arriving signal. Typically pulsars lose energy and their rotation frequency decreases. If pulsar signals are to be used to try to detect gravitational waves, then the physical effects of the pulsars themselves must be well understood. One must account for dispersion of the signal by the interstellar medium, and also account for fluctuations in the dispersion. The period derivative of the pulsar, caused by the loss of rotational energy via the emission of gravitational waves, must be included. The exact location of the pulsar in the sky, along with its proper motion, must be known to high precision [315].

After the discovery of the first pulsar, and subsequent detections of others, it was observed that some pulsars, such as PSR 1937 + 21, could be as stable as atomic clocks [315]. For this pulsar the frequency stability was observed to be $\Delta f/f \sim 6 \times 10^{-14}$ when averaged over times longer than 4 months. With the observations of this pulsar, and through the observed frequency stability, it was possible in 1987 to set a limit on the energy density of the stochastic background to be $\Omega_{\text{GW}}(f) h^2 < 4 \times 10^{-7}$ at a frequency of $7 \times 10^{-7} \text{ Hz}$ [315]. During this early period in pulsar observations many quickly used their observations to also constrain the stochastic background. For example, Hellings and Downs [316] used the observations from four pulsars to constrain the stochastic background to $\Omega_{\text{GW}}(f) h^2 < 1.4 \times 10^{-4}$ at frequencies around 10^{-8} Hz .

Using pulsar timing to try to observe gravitational waves is currently a very active research area, involving numerous collaborations around the world [317–322]. The current observations concentrate on signals with frequencies in the range of 10^{-9} – 10^{-7} Hz [24]. A stochastic background is the most likely signal source for the current pulsar timing experiments, namely the background produced by all of the inspiral and mergers of super massive black hole binaries over the history of the universe [24, 323].

The European Pulsar Timing Array has recently reported limits on the stochastic background based on the observation of six pulsars over 18 yr. Their upper limits on the energy density of the stochastic background is $\Omega_{\text{GW}} h^2 < 1.1 \times 10^{-9}$ at 2.8 nHz. This limit places stringent constraints on the super-massive binary black hole population in the universe. This analysis also constrains the string tension to $G\mu < 1.3 \times 10^{-7}$ for a Nambu–Goto field theory cosmic string network [324].

NANOGrav [129] has reported the results from an examination of nine years of pulsar data involving 37 pulsars [129]. The upper limit on the energy density of the stochastic background was reported to be $\Omega_{\text{GW}} h^2 < 4.2 \times 10^{-10}$ at frequency $3.3 \times 10^{-9} \text{ Hz}$ [129]. These results were then improved with the goal of constraining cosmic string parameters [295]. Using a new analysis of the NANOGrav results, a constraint has been found on the cosmic string tension of $G\mu < 1.5 \times 10^{-11}$ [295].

The Parkes Pulsar Timing Array (PPTA) [318, 325] uses the Parkes 64 m radio telescope to observe 24 pulsars. With this data they have constrained the energy density of the stochastic background to be $\Omega_{\text{GW}} < 2.3 \times 10^{-10}$ at 6.3 nHz for a spectral index of $n_t = 0.5$ [169]. The limit for a spectral index of $n_t = 0$ is the same to two decimal places [45]. It is expected that with five subsequent years of data the PPTA could achieve a limit of $\Omega_{\text{GW}} < 5 \times 10^{-11}$, but that will be even further improved by combining the results from the different pulsar timing collaborations [45] as part of the International Pulsar Timing Array [321].

3.10. Doppler tracking limits

The same techniques that are applied to radio signals from pulsars for the detection of gravitational waves can be applied

to signals transmitted from spacecraft traveling through our solar system [326, 327]. In fact, the Doppler tracking of spacecraft was considered and analyzed before pulsar timing [328]. Originally intended to look for gravitational waves emitted from pulsars, the Doppler tracking technique is also applicable to searches for a stochastic background of gravitational waves. The Earth and the spacecraft are considered as free masses. A limit can be placed on the energy density of the stochastic background in the frequency range of 10^{-6} – 10^{-2} Hz [329]. Signals from many different spacecraft have been used, including the Viking [330], Voyager [331], Pioneer 10 [332], Pioneer 11 [333], and Cassini [334–336]. The best upper limit on the energy density of the stochastic background comes from the analysis of the Cassini data, giving $\Omega_{\text{GW}} < 0.025$ at a frequency of 1.2×10^{-6} Hz and assuming a value for the Hubble constant of $75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [327, 336]. Using the currently accepted value of $H_0 = 67.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [44] this reduces the limit to $\Omega_{\text{GW}} < 0.03$.

3.11. Cosmic microwave background anisotropy limits

The near isotropy of the cosmic microwave background (CMB) can be used to constrain the energy density of the stochastic background at very low frequencies. There are two ways in which gravitational waves will disturb the CMB. Gravitational waves today with wavelengths of the order of the horizon size will produce a quadrupole anisotropy, while gravitational waves at the time of recombination will cause fluctuations on smaller angular scales that can be observed today [337]. In figure 3 the curve labeled CMB corresponds to the limits on $\Omega_{\text{GW}}(f)$ from the CMB measurements of the Planck satellite [338]. An energy of gravitational waves above this level would have changed the observations made on the CMB [339–341], such as those made by Planck [338].

3.12. Indirect limits

The production of deuterium, helium and lithium in the early universe can be used to constrain the energy density of the stochastic background. This Big Bang nucleosynthesis (BBN) limit provides an important constraint on the stochastic background. If the energy density of the gravitational waves is too large when these light nuclei are produced, the abundances today would be different from what is actually observed [342]. Too much gravitational wave energy would speed up the universe's expansion rate, thereby reducing the amount of helium formed from deuterium, altering the observed ratios.

The baryon density in the universe today is in the range of $\rho_{\text{b}} = (3.9\text{--}4.6) \times 10^{-31} \text{ g cm}^{-3}$. This then translates into a relationship with the critical density of the universe, namely $\Omega_{\text{b}} = \rho_{\text{b}}/\rho_{\text{crit}} = 0.046\text{--}0.053$. The majority of the baryon mass of the universe is made up of neutral hydrogen. The primordial mass fraction of helium ${}^4\text{He}$ is $Y_{\text{p}} = \rho({}^4\text{He})/\rho_{\text{b}} \approx 0.25$. The primordial mass fraction for deuterium D and helium ${}^3\text{He}$ are of the order 10^{-5} , while for Lithium ${}^7\text{Li}$ it is at the 10^{-10} level [343]. The observations of the mass ratios for primordial nucleosynthesis limit the energy density

of gravitational waves to $\Omega_{\text{GW}} < 1.8 \times 10^{-5}$ for frequencies in excess of 10^{-10} Hz [128, 308, 344].

Observations of the CMB, BBN, and baryon acoustic oscillations (BAO) [345] can be combined to provide a limit on the energy density of the stochastic background [45, 96, 128]. It can be shown that an upper limit on the energy density of the stochastic background for frequencies above 10^{-15} Hz can be made with

$$\Omega_{\text{GW}} \leq \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} (N_{\text{eff}} - 3.046) \Omega_{\gamma} \quad (23)$$

where the energy density of the CMB is $\Omega_{\gamma} = 2.473 \times 10^{-5}/h^2$ [128]. The term N_{eff} is the effective number of neutrinos, and the measurements of the Z boson width [343] limit its value. Studies considering the behavior of the three neutrino families in the early universe give a value of $N_{\text{eff}} \approx 3.046$ [346]. The presence of a large energy density of gravitational waves would alter the value of N_{eff} observed via cosmological observations today. Combining equation (23) with the value of Ω_{γ} implies

$$\Omega_{\text{GW}} h^2 \leq 5.6 \times 10^{-6} (N_{\text{eff}} - 3.046), \quad (24)$$

which can then be used to limit Ω_{GW} based on BBN, CMB and BAO observations [128]. Recent observations place a limit of $\Omega_{\text{GW}} \leq 3.8 \times 10^{-6}$ [45, 96].

3.13. B-modes in the cosmic microwave background

The CMB holds much information pertaining to a stochastic background produced at the earliest moments of the universe. For example, the gravitational waves produced during inflation should leave their imprint on the CMB when it was produced 3.8×10^5 yr after the Big Bang; this is the recombination time when the temperature of the universe was $\sim 3 \times 10^3$ K.

As described above, quantum fluctuations during inflation will create a stochastic background of gravitational waves. Density fluctuations will also be created. Both of these can affect the polarization content of the CMB. However, they can be differentiated from one another, namely by breaking down the composition of the CMB polarization into a curl-free component (an E mode), and a curl component (a B mode) [347]. The presence of gravitational waves produced during inflation would be responsible for introducing B modes into the CMB polarization at the time of recombination. Gravitational waves can also induce fluctuations in the temperature of the CMB. An excellent summary of all aspects of B modes is presented in [347].

Gravitational waves affect the metric of spacetime, which can then consequently affect a photon's energy. At the time of recombination the gravitational waves and the photons were traveling within the cosmic fluid of material present at the time, mostly protons, electrons and neutrinos. Of course there is also a change in the energy of the photons due to the expansion of the universe. The presence of gravitational waves alone does not affect the polarization of the photons, only their energy. Similarly, density fluctuations in the cosmic fluid will induce a gravitational redshift in the photons, but

not affect their polarization. However, as photons Thomson scatter off of the electrons present, a net polarization can be induced.

The measure for the amount of gravitational waves produced during inflation is typically expressed in terms of the tensor to scalar ratio,

$$r = \frac{\Delta_h^2}{\Delta_R^2}, \quad (25)$$

where Δ_h^2 is the gravitational wave power spectrum and Δ_R^2 is the curvature power spectrum. The r value can also be directly related to the potential of the inflaton, ϕ , during inflation, namely $V(\phi)$; see [347] for details.

Unfortunately gravitational waves are not the only means to create B modes in the polarization of the CMB. Gravitational lensing of the CMB can also produce B modes. This would be caused by massive objects between us (as observers) and the surface of last scattering of the CMB [347–349]. This effect has been observed [350–352]. However, with the present knowledge of the parameters describing our universe, Λ CDM, it is possible to accurately predict the amount of B modes in the CMB polarization created by lensing. The influence of gravitation waves on the B modes will be prominent in the spherical harmonic range from $\sim l = 10$ to $\sim l = 100$, or roughly an angular scale of $\sim 0.1^\circ$ to $\sim 1^\circ$ [347].

The most serious obstacle to directly observing the effects of gravitational waves on the CMB is the presence of the material in and about our galaxy. Synchrotron emission in the galaxy is a foreground which will contaminate CMB polarization studies for photon frequencies under 100 GHz [347, 353]. Dust grains tend to align themselves with the galactic magnetic field; the thermal emission from these grains tends to be polarized [354]. The presence of the material makes the search for B modes in the galactic plane impractical, and hence observations need to take place at high galactic latitudes [347].

When observations are made of the polarization of the CMB across a patch of sky, a decomposition can be made of the E modes and B modes. The polarization power as a function of angular scale (or exactly, spherical harmonic number l) is measured and plotted. From that the tensor to scalar ratio, r , can be extracted [347]. Numerous observation teams are currently attempting to find the B modes produced by gravitational waves. In 2014 the BICEP2 Collaboration claimed an observation of B modes in the range $30 < l < 150$, or roughly 0.3° – 1.5° [355]. However, the results were quickly challenged [356], and subsequent analyses showed that the observed B modes were actually due to galactic dust, and reported an upper limit of $r < 0.12$ at 95% confidence [357]. Subsequent observations by BICEP2 and the KECK Array have further reduced this limit to $r < 0.09$ at 95% confidence; combining the results with Planck CMB temperature data and baryon acoustic oscillation results further constrains the ratio to $r < 0.07$ at 95% confidence [358]. There are other attempts by other groups to observe or constrain the B modes due to gravitational waves [359–362].

3.14. Normal modes of the Earth, Moon and Sun

The measurement of the normal modes of the Sun, Earth and the Moon have been used to limit the energy density of the stochastic gravitational wave background. The idea of using the Earth itself as a gravitational wave detector goes back to 1969 with a proposal from Freeman Dyson [363]. The application of actual data pertaining to motions of the Sun and Earth started as early as 1984 when Boughn and Kuhn [364] analyzed the process by which a gravitational wave background drives the normal modes of a spherical body. Using data of the observed line of sight velocity of the surface of the Sun they were able to constrain $\Omega_{\text{GW}}(f)$ to be less than 100 at a frequency of 4×10^{-4} Hz. The Earth's cross-section to the background of gravitational waves is smaller than the Sun's because the Earth is much smaller. However, the data on seismic activity is much better for the Earth. The limit achieved from the Earth data was also $\Omega_{\text{GW}}(f) < 100$ at frequencies of 2×10^{-3} Hz and 2×10^{-2} Hz.

Much progress has subsequently been made with these types of studies. Recent observations of the Sun have used helioseismology. A stochastic background of gravitational waves would excite stars like our Sun, causing them to oscillate. For the Sun, high precision radial velocity data is used to monitor the motion. Specifically, limits on the the high frequency quadrupolar g modes [365] are used to constrain the stochastic background. A model of the sun has been used where it is assumed to be a spherical body with a negligible shear modulus. The best constraint with this method is $\Omega_{\text{GW}} < 4.0 \times 10^5$ at 0.171 mHz [366].

The method of Dyson [363] using the Earth to attempt to measure gravitational waves was implemented using seismometer data [367]. Correlations were made between pairs of seismometers. The seismometers used in this study were located around the world. The surface of the Earth was considered to be a free and flat surface in its response to gravitational waves. The limit derived was $\Omega_{\text{GW}} < 1.2 \times 10^8$ in the 0.05–1 Hz band using one year of data [367].

This study was then extended to take into account the internal structure of the Earth [97]. This allowed for lower frequencies to be addressed, since below 50 mHz there is global coherence in the seismic motion. The new study used both the data from gravimeters and a model of the response of the Earth's modes to gravitational waves. Ten years of data from the superconducting gravimeters for the Global Geodynamics Project [368] were analyzed. For frequencies between 0.3 mHz and 5 mHz, limits were placed on the energy density of the stochastic background, Ω_{GW} , with the limits ranging between 0.035 and 0.15 [97].

Seismic arrays on the moon have also been used to limit the energy density of the stochastic background [369]. Seismometers were placed on the moon between 1969 and 1972 as part of the Apollo 12, 14, 15 and 16 missions. Data was acquired until 1977. The seismic noise on the moon is less than that on Earth. From the lunar seismometer data the integrated energy density of the stochastic background from 0.1 to 1 Hz can be constrained to $\Omega_{\text{GW}} < 1.2 \times 10^5$ [369]. This is currently the best limit in this frequency band.

4. Conclusions

The observations of gravitational waves by Advanced LIGO [5, 6] and Advanced Virgo [12] have created tremendous excitement in the world of physics [4, 8–11, 13, 14]. In addition to signals from the coalescence of binary black hole and binary neutron star systems, numerous other types of signals are expected [283]. One of those is a stochastic background of gravitational waves. The observations of Advanced LIGO and Advanced Virgo predict that these instruments, in the coming few years, should detect a stochastic background created by all binary black hole and binary neutron star mergers throughout the history of the universe [17, 18, 51]. It is also possible that in the coming years LIGO and Virgo could detect a stochastic background from other sources, for example from cosmic strings [142]. The observations by the Advanced LIGO–Advanced Virgo network will likely be made in the 20–100 Hz band.

In the coming years it is likely that pulsar timing could make an observation, most likely of a stochastic background. This would be the stochastic background produced by all of the inspiral and mergers of super massive black hole binaries over the history of the universe [24, 323]. The frequency band for these observations would be 10^{-9} – 10^{-7} Hz. Numerous collaborations around the world are attempting to detect gravitational waves, and especially the stochastic background [317–322].

It is also probable that in the coming years the imprint made on the polarization of the CMB by gravitational waves created by quantum fluctuations during inflation will be measured. Observations by BICEP2 and the KECK Array have set a limit on r , the tensor-to-scalar ratio, of $r < 0.09$ at 95% confidence; when the results are combined with Planck CMB temperature data and baryon acoustic oscillation results the constraint is narrowed to $r < 0.07$ at 95% confidence [358]. Galactic dust is a continual problem in the quest to observe the effect of primordial gravitational waves [356]; however, observing the CMB at multiple frequencies may allow the effects of the dust to be disentangled if r is not inordinately small. Many groups are trying to observe or constrain the B modes due to gravitational waves [359–362].

Future gravitational wave detectors will offer exciting prospects for observing the stochastic background. Third generation ground-based gravitational wave detectors, such as the Einstein Telescope [194] or the Cosmic Explorer [195], will have better sensitivity by a factor of ≈ 10 than the target sensitivity of Advanced LIGO or Advanced Virgo. An exciting prospect for these detectors is that they should be able to directly observe almost every stellar mass binary black hole merger in the observable universe. This could allow them to directly detect and remove from the stochastic search the astrophysical foreground. By removing this foreground the third generation detection detectors could be sensitive to a cosmologically produced background at the $\Omega_{\text{GW}} \sim 10^{-13}$ level with 5 yr of observations [196]. This will then bring the third generation detectors into a sensitivity regime for important cosmological observations.

The LISA mission has been accepted by ESA, with contributions to be made by NASA [31, 32]. The current plan is for a 2034 launch, with a mission lasting 4 yr, with the possibility of an extension to 10 yr of total observation time. While a major goal of LISA will be to observe a cosmologically produced stochastic background, there will be a significant astrophysically produced foreground that will make this task difficult. For example, galactic binaries will mask other more interesting signals, and different techniques have been proposed for addressing the galactic binary signals [199, 200, 220–222, 294]. The detection of gravitational waves from binary black hole inspirals implies that there will be a stochastic background from these systems [17], and this stochastic background will also be observable by LISA. If the astrophysical foreground can be addressed, LISA could potentially have a sensitivity to a stochastic background at the $\Omega_{\text{GW}}(f) \approx 10^{-12}$ level in the 10^{-4} Hz to 10^{-1} Hz band. This sensitivity could allow LISA to observe the consequences, for example, of a first-order electroweak phase transition [153, 154], or of the presence of cosmic strings [295].

The recent detection of gravitational waves is the start of a new era. The stochastic gravitational wave background will hold information on some of the most important events in the history of the universe. In the coming years we can expect this background to be observed, and stunning revelations about the universe should be discovered.

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