Heliyon 8 (2022) e11668

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

Example of negative energy density in a classical electron model

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HIGHLIGHTS

- Classical model for the electron.
- Electron mass from electromagnetic energy.
- Region of negative energy density.
- Poincaré stresses.

ARTICLE INFO

Keywords:
Classical electron models
General relativity
Negative energy density

ABSTRACT

Classical models of the electron have been predicted to have negative rest energy density in certain regions. Using the model of the electron by Blinder we show that there are regions containing negative energy density, although the integral of the energy density over all space gives the electron rest mass. If the spin of the electron is ignored, then all regions of space have positive energy density with the Blinder model. The existence of Poincaré stress for the Blinder model is also demonstrated. The classical model for the electron discussed here admittedly does not involve quantum electrodynamics, where the infinite self energy is made finite with renormalization methods.

1. Introduction

The structure of the electron has been an active topic of research since its discovery [1, 2]. An important question is how to avoid an infinite amount of energy in the electromagnetic field, and this has been a subject of much research over the years [3, 4, 5, 6, 7]. Certainly quantum electrodynamics addresses a finite mass and charge for the electron. Many studies have tried to associate the electromagnetic energy of the electron with its rest mass [2, 8, 9].

Previous research efforts have addressed the question of classical models of the electron containing negative energy density [9, 10, 11, 12, 13]. In order to keep the electromagnetic energy finite it typically requires regions of negative energy density. This has also introduced the concept of Reissner-Nordström repulsion [14, 15, 16]. Negative energy density and the expansion of space-time is critical for the cosmological theory of inflation [17, 18].

Here we address the classical model of the electron by Blinder [19, 20, 21], and show that it contains a region of negative energy density. If the spin of the electron is set to zero, then the Blinder model gives a positive energy density for all space. Note that there is no singularity in the Blinder model of the electron, and the space-time curvature is always finite. Associations have been previously made between singularities and negative energy density in models for the electron [14, 22]. Clearly classical models for the electron neglect the demonstrated success of quantum electrodynamics and renormalization to ensure that the electron mass is finite. However interesting comparisons can still be made with these classical models, especially when considering the limit when Planck's constant goes to zero.

One can use general relativity and the Reissner-Nordström metric to describe space-time about a massive point charge. The integration of the electric field energy diverges as the radial coordinate goes to zero, as displayed in Eq. (6) below. This is also true for the Kerr-Newman metric where the massive point charge also has spin. The energy density is positive definite for all space with the Kerr-Newman metric. The model of Blinder attempts to avoid the infinite electromagnetic energy [19, 20, 21]. However, as displayed below, it does contain a region of negative mass energy and has a non-electromagnetic Poincaré stress.

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https://doi.org/10.1016/j.heliyon.2022.e11668
Received 7 August 2022; Received in revised form 22 October 2022; Accepted 10 November 2022

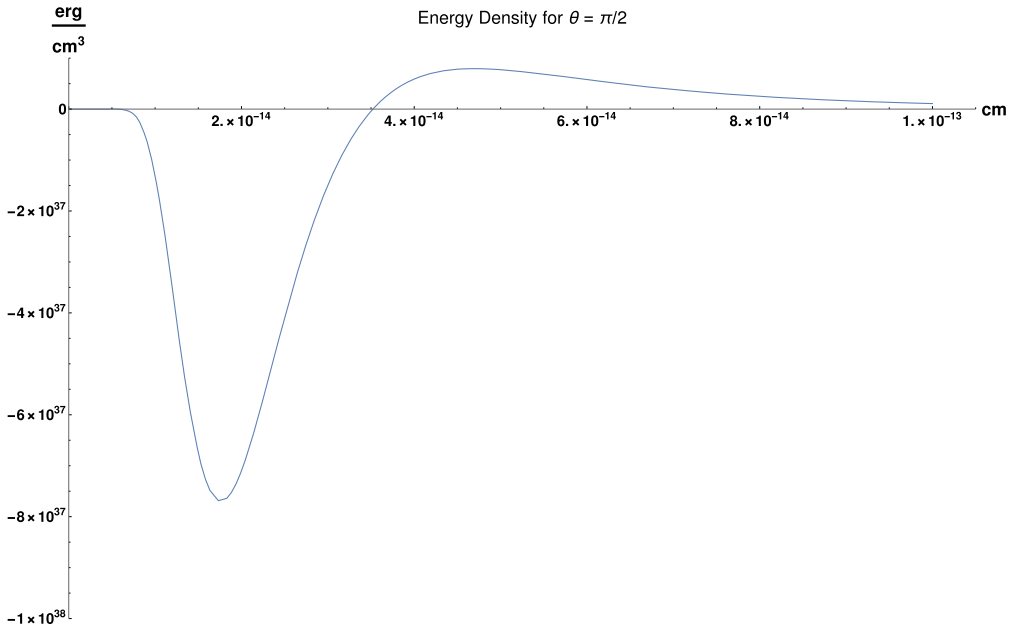

Fig. 1. The energy density T_0^0 for the Blinder model, Eq. (8), for $\theta = \pi/2$.

The remainder of the paper is organized as follows. In Section 2 we review Blinder's classical model for the electron [21]. This is a modification to the Kerr-Newman metric. With the new metric it is shown that the total electromagnetic energy and angular momentum (spin) matches that of the electron. Section 3 presents the concept of negative energy density. With the Blinder model there are regions of space containing a negative energy density. The Ricci scalar for the Blinder model is also presented, and shown to be finite everywhere. The concept of non-electromagnetic stresses to counteract the Coulomb repulsion in the classical electron model is presented in Section 4. A conclusion is given in Section 5.

2. Blinder model

We follow the notation of [21] and use cgs units. The electron has a mass m, charge e_q, and spin $s = \hbar/2$. With Newton's constant G and speed of light c we define $M = Gm/c^2$, $Q = \sqrt{G}e_q/c^2$, and $S = Gs/c^3$. One can use the Kerr-Newman metric (Eqs. (1), (2), (3) and (4)) to write [23]

$$ds^2 = \frac{\Delta}{\rho^2}[dt - asin^2\theta d\phi]^2 - \frac{sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - adt]^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 \ , \tag{1}$$

where

$$\Delta = a^2 + r^2 - 2Mr + Q^2 \ , \tag{2}$$

$$\rho^2 = r^2 + a^2 cos^2\theta \ , \tag{3}$$

and

$$a = \frac{S}{M} = \frac{\hbar}{2mc} \ . \tag{4}$$

The energy density [24] is given by

$$T_0^0 = \frac{e^2(a^2 + r^2 + a^2 sin^2\theta)}{8\pi(r^2 + a^2 cos^2\theta)^3} \ . \tag{5}$$

Using Eq. (5), the total energy in the electromagnetic field

$$W = \int T_0^0 \rho^2 sin\theta dr d\theta d\phi \tag{6}$$

diverges as $r \to 0$.

In the model of Blinder [19, 20, 21] one changes the definition of Δ, namely

$$\begin{aligned} \Delta &= a^2 + r^2 - 2Mre^{-Q^2/2Mr} \\ &= a^2 + r^2 - 2Mre^{-r_0/2r} \ , \end{aligned} \tag{7}$$

where the classical electron radius is $r_0 = e_q^2/mc^2 = 2.818 \times 10^{-13}$ cm. This produces only a very small change in the metric compared to the Kerr-Newman metric, except for the smallest distances. It also renders the total electromagnetic energy to be finite and equal to the electron rest mass. For $r > r_0$ the electric and magnetic fields match what one expects for the electron. For smaller radii the fields are more complex and phenomenologically resemble vacuum polarization [19, 20, 21]. With the Blinder model the energy density is

$$\begin{aligned} T_0^0 = &\frac{a^2 e_q^2 e^{-\frac{r_0}{2r}}}{8\pi\left(a^2\cos^2\theta + r^2\right)^3} + \frac{e_q^2 r^2 e^{-\frac{r_0}{2r}}}{8\pi\left(a^2\cos^2\theta + r^2\right)^3} \\ &+ \frac{a^2 e_q^2 \sin^2\theta e^{-\frac{r_0}{2r}}}{8\pi\left(a^2\cos^2\theta + r^2\right)^3} - \frac{a^2 e_q^2 r_0 \sin^2\theta e^{-\frac{r_0}{2r}}}{32\pi r^3\left(a^2\cos^2\theta + r^2\right)^2} \ . \end{aligned} \tag{8}$$

With this the total electromagnetic energy is equal to the rest mass of the electron,

$$W = \int T_0^0 \rho^2 sin\theta dr d\theta d\phi = mc^2 \ . \tag{9}$$

Note that the volume element $\rho^2 sin\theta dr d\theta d\phi$ is the same and independent of Δ. Note too that

$$\begin{aligned} T_3^0 = &\frac{a^3 e q^2 r_0 \sin^2\theta e^{-\frac{r_0}{2r}}}{32\pi r^3\left(a^2\cos^2\theta + r^2\right)^2} + \frac{a e_q^2 r_0 \sin^2\theta e^{-\frac{r_0}{2r}}}{32\pi r\left(a^2\cos^2\theta + r^2\right)^2} \\ &- \frac{a^3 e_q^2 \sin^2\theta e^{-\frac{r_0}{2r}}}{4\pi\left(a^2\cos^2\theta + r^2\right)^3} - \frac{a e_q^2 r^2 \sin^2\theta e^{-\frac{r_0}{2r}}}{4\pi\left(a^2\cos^2\theta + r^2\right)^3} \ . \end{aligned} \tag{10}$$

From Eq. (10), the component of spin about the z-axis is

$$s_z = \frac{1}{c}\int T_3^0 \rho^2 sin\theta dr d\theta d\phi = \frac{\hbar}{2} \ . \tag{11}$$

Hence with the Blinder model the electromagnetic field can describe the rest mass and spin of the electron.

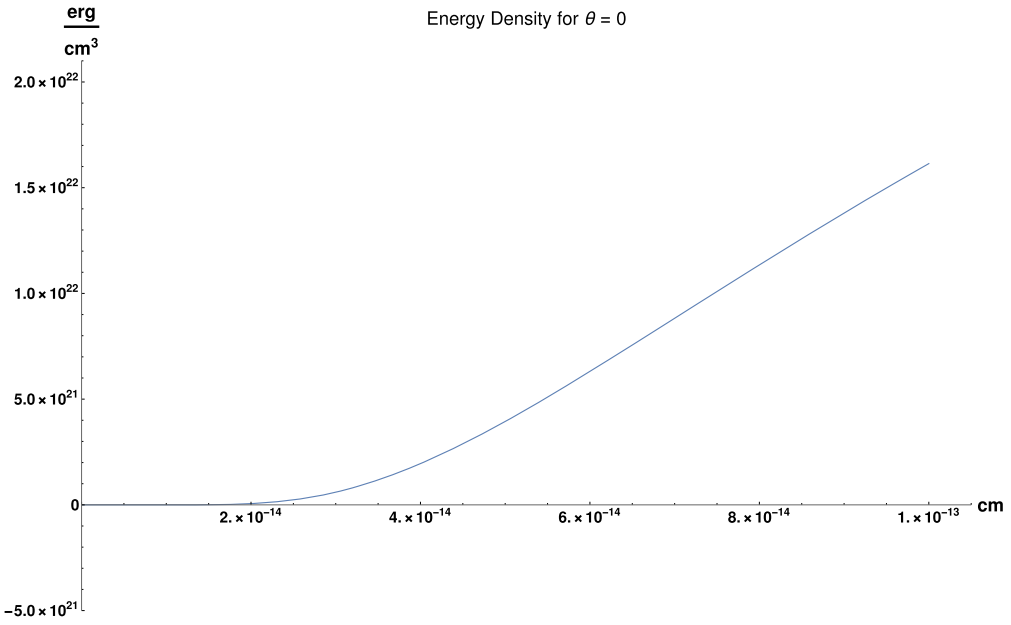

Fig. 2. The energy density T_0^0 for the Blinder model, Eq. (8), for $\theta = 0$. This is displayed over the same radii as in Fig. 1.

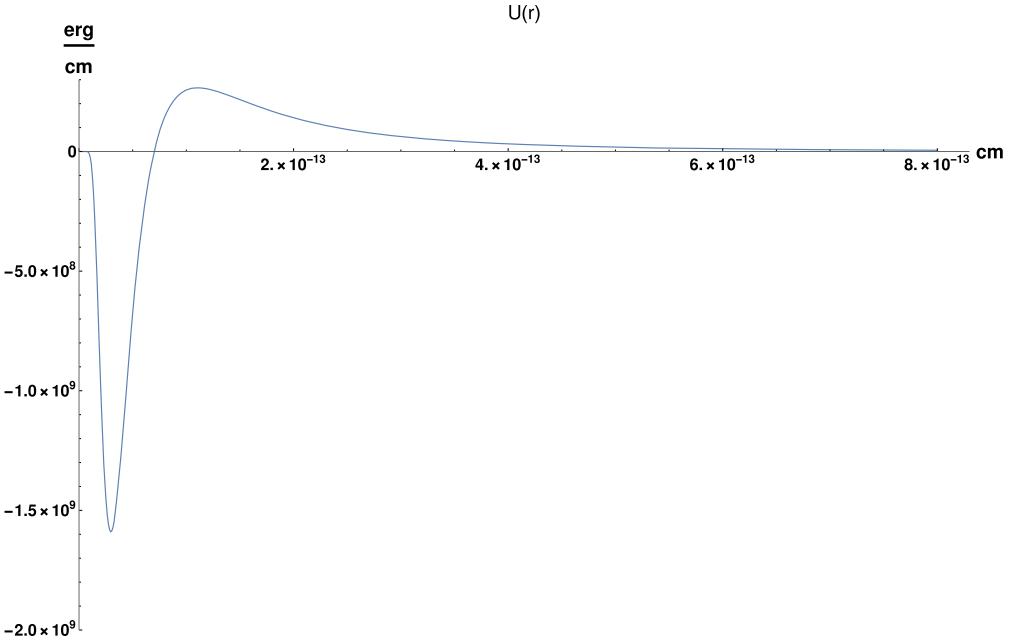

Fig. 3. The energy density T_0^0, integrated over ϕ and θ, namely $U(r)$ for the Blinder model, Eq. (12).

3. Negative energy density

We now examine the energy density T_0^0 in the Blinder model, that when integrated over all space gives the electron rest mass. It is interesting to note that a region, near to $r = 0$, actually has a negative energy densities, similar to what has been discussed in [9, 10, 11, 12, 13]. This is demonstrated in Fig. 1, which displays T_0^0 for the Blinder model, Eq. (8), for $\theta = \pi/2$. As the angle θ decreases the amount of negative energy density also decreases. Negative energy density exists for all angles except for $\theta = 0$, for which is entirely positive. This is displayed in Fig. 2. Note that if there is no spin ($a = 0$) the energy density represented by Eq. (8) is positive definite over all space.

One can also view the extent of the negative energy density region by integrating T_0^0, Eq. (8), over ϕ and θ. We define

$$U(r) = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta T_0^0 \rho^2 sin\theta \ . \tag{12}$$

This is displayed for the electron in Fig. 3. The negative energy density extends from $r = 0$ to $r = 7.01 \times 10^{-14}$ cm $\approx r_0/4$. The total energy density in this region is $-57.9\ mc^2$, where $mc^2 = 8.187 \times 10^{-7}$ erg for the electron. The region from $r = 7.01 \times 10^{-14}$ cm $\approx r_0/4$ to $r = 1.40 \times 10^{-12}$ cm $\approx 4.96\ r_0$ compensates with a total energy of $+57.9\ mc^2$. It is then the integration from $r = 1.40 \times 10^{-12}$ cm $\approx 4.96\ r_0$ to infinity that gives the rest mass energy of the electron, $mc^2 = 8.187 \times 10^{-7}$ erg.

Note that with the Blinder model the presence of negative energy density cannot be linked to the presence of a singularity [22], as there is none. The exponential term in Eq. (7) is responsible for keeping the curvature finite, even at $r = 0$. For the Blinder model the Ricci scalar is

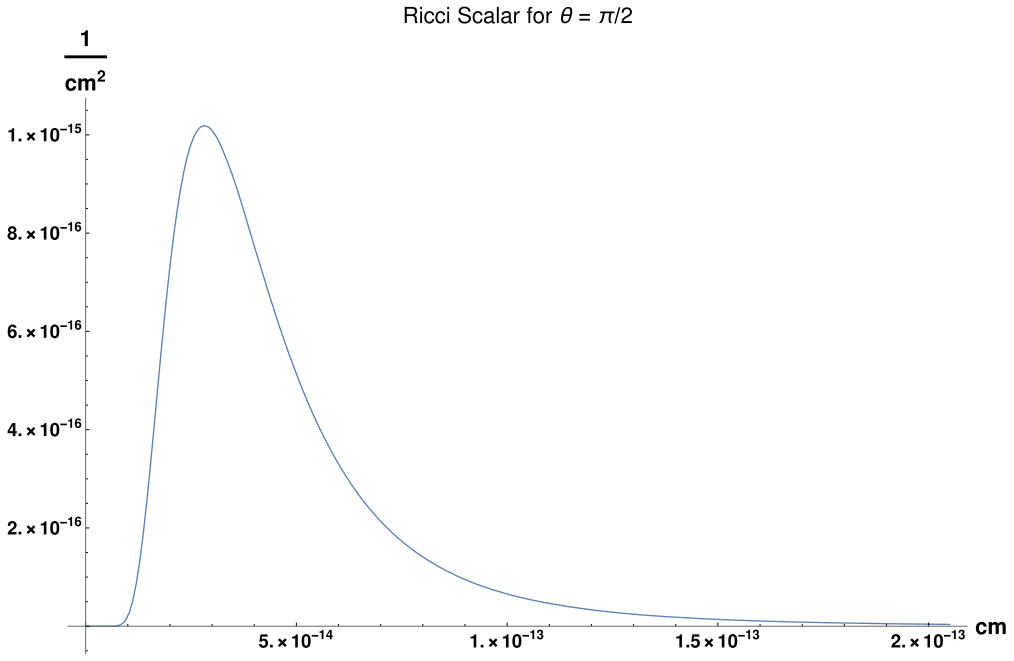

Fig. 4. The Ricci scalar for the Blinder model, R Eq. (13), for $\theta = \pi/2$.

$$R = \frac{G}{c^4} \frac{e_q^2 r_0 e^{-\frac{r_0}{2r}}}{2r^3 \left(a^2 \cos^2\theta + r^2\right)} \,. \tag{13}$$

This is displayed in Fig. 4. The peak for the Ricci scalar is in the same region for which the energy density is negative, as displayed in Fig. 1 and Fig. 3. Note that as $r \to 0$, $R \to 0$.

Negative energy density has been previously discussed in the context of Reissner-Nordström repulsion [14, 15, 16]. Various models for the electron have regions of negative energy density and hence gravitational repulsion. The negative energy density creates an equilibrium with the positive energy density of the electromagnetic field. As displayed here, this is the case for the electron model of Blinder as well.

4. Stresses

In the Blinder model for the electron the trace of the stress energy tensor, Eq. (14), is non zero, specifically [21],

$$T^{\mu}_{\mu} = \frac{e_q^2 r_0 e^{-\frac{r_0}{2r}}}{16\pi r^3 \left(a^2 \cos^2\theta + r^2\right)} \,. \tag{14}$$

This implies that there is some other stress-energy along with that of the electromagnetic field. If one considers an imponderable perfect fluid, $\rho = 0$, with no radial pressure, $p_r = 0$, then the tangential pressure could be given as

$$p_\theta = p_\phi = \frac{e_q^2 r_0 e^{-\frac{r_0}{2r}}}{32\pi r^3 \left(a^2 \cos^2\theta + r^2\right)} \,. \tag{15}$$

Blinder proposes that this could be considered as a classical counterpart to the effects of vacuum polarization [21]. A similar result would occur for a perfect fluid with $\rho = -p_r$ [25]. This argument has also been proposed by Grøn [16]. The concept of non-electromagnetic stresses to counteract the Coulomb repulsion in an electron model dates back to Poincaré [26, 27]. This Poincaré stress has been addressed more recently as well in models for the electron [15, 16]. For the Blinder model, the stresses represented by Eq. (15) are similar to the Poincaré stress.

This tangential pressure is displayed in Fig. 5 for $\theta = \pi/2$. Note that the peak for the tangential pressure is in the same region for which the energy density is negative, as displayed in Fig. 1 and Fig. 3.

5. Conclusion

The classical model of the electron of Blinder [19, 20, 21] has a number of interesting features. The metric is a slight variation from the Kerr-Newman metric. The total energy density sums to the rest mass of the electron, mc^2 (see Eq. (9)), while the total z-component of spin sums to $\hbar/2$ (see Eq. (11)). The Blinder model has regions of negative energy density, similar to the Reissner-Nordström repulsion in other classical models of the electron [14, 15, 16]. The Ricci scalar is also shown to be finite everywhere, including $r = 0$, for the Blinder model. The exponential term in the Blinder metric, Eq. (7), suppresses any singularity. In order compensate for the Coulomb repulsion, a Poincaré stress is present (see Eq. (15)). Quantum electrodynamics provides the comprehensive description of the electron and its electromagnetic field. However, the Blinder model and general relativity combine to provide an interesting classical description of the electron, its mass, its spin, and its electromagnetic field.

Declarations

Author contribution statement

Nelson Christensen, Ph.D: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availability statement

No data was used for the research described in the article.

Declaration of interests statement

The authors declare no conflict of interest.

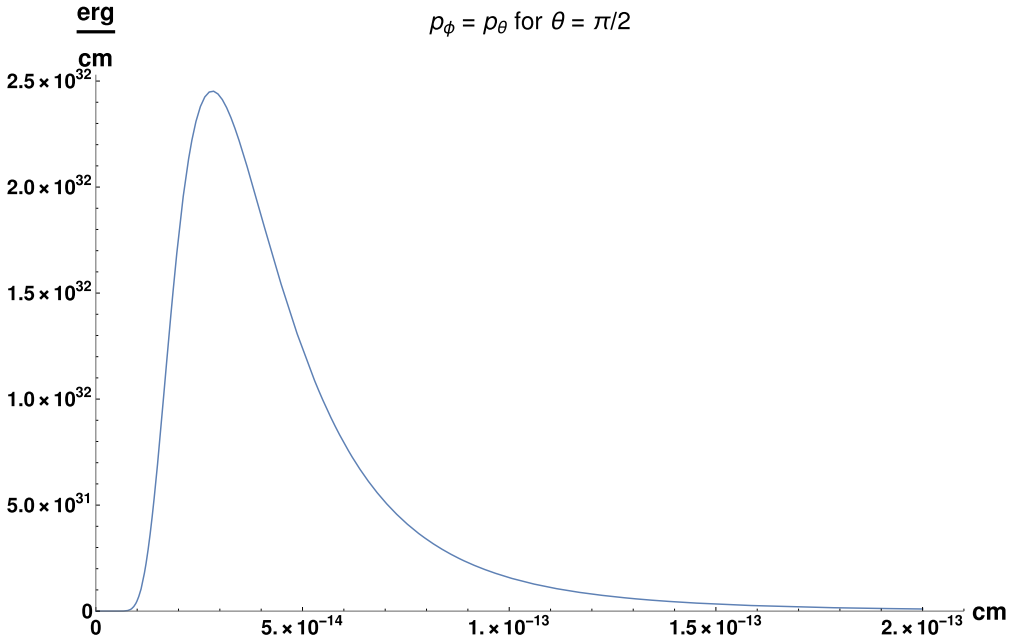

Fig. 5. The tangential pressure for the Blinder model, $p_\theta = p_\phi$ Eq. (8), for $\theta = \pi/2$.

Additional information

No additional information is available for this paper.

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